Essays on improving estimation of risk with application to stocks and options Introduction

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Abstract

Risk has always been a topic of interest for academicians and practitioners (managers and retail investors) alike. It has regained importance in the financial world after the heavy losses that the investors had to incur during the recent financial crisis of 2008. Quantifying risk has always been a challenge. While Value-at-Risk (VaR) has been more or less universally accepted as a risk measure, there are researchers who are sceptical about using VaR particularly for a longer risk horizon. Value-at-Risk can be defined as the maximum loss that may be incurred by a portfolio at the end of a specified period at a given significance level. In other words, it represents a quantile of an estimated profit and loss (return) distribution. It is a very simple and powerful measure of risk as it indicates the threshold of the maximum loss that may be incurred. While being popular in the industry for its simplicity and ease in implementation, VaR suffers from a number of drawbacks. One of the major complications in measuring VaR arises when the portfolio under consideration has a non-linear payoff structure such as options. In such cases, the VaR cannot be computed directly from the risk factor distributions. Instead, a profit and loss (P&L) distribution for the option needs to be constructed from the risk factors in order to compute the VaR. Another major limitation of VaR is that it gives the risk exposure of the portfolio at the end of the period. It provides no information about the risk that the portfolio might be exposed to during the risk horizon. This is where MaxVaR comes into the picture. MaxVaR is defined as the maximum loss that may be incurred by a portfolio during a given period at a given significance level. It is also known as intra-horizon risk. The importance of MaxVaR can be gauged from the fact that it provides information about the interim risk that our portfolio is exposed to as compared to VaR which provides information only about the risk exposure of the portfolio at the end of the risk horizon. This could be really important from a regulator's point of view. Even for a retail investor, in a mark to market environment, it is a useful tool as it can hint towards possible margin calls during the holding period and give the investor an opportunity to act accordingly. The essays in this thesis deal with various issues related to risk quantification and estimation. Our main aim is to quantify and estimate risk in a better way. While the first three essays are developed in the context of options, a special class of financial instruments, the final essay is on the risk observed in equity markets.The first essay of the thesis looks at Diffusion Processes and First Passage Time Probabilities in general. Specifically, for a Pearson Diffusion Process, we find the lower and upper bounds of the First Passage Time density and show that the density may be approximated by the upper bound with an error of approximately 5%. We then show that the profit and loss function of the options can be assumed to follow a Pearson Diffusion Process and based on the results obtained earlier on First Passage Time Probabilities, we can obtain the MaxVaR for the options, which can serve as a risk measure. For robustness, we extend our study to various index options. In our second essay, we estimate the static VaR for a portfolio of options using Pearson Type-IV and skewed Student's t distribution. These distributions are used as they can fit the P&L (profit and loss function) of the options better, which is fat-tailed and skewed. The estimates so obtained for VaR are better than the existing methods. We also

estimate the dynamic VaR for the options by fitting a time-varying mean and volatility model to the P&L of the options. The VaR estimates so obtained are tested using standard back testing methods and are found to be efficient. We also present a comparison of several existing methods (methods that are most commonly used and easy to comprehend and implement are selected for comparison) of calculating VaR for options. For robustness, we extend our study to various other index options as well as repeat the study for in-the-money and at-the-money options as well. We conclude that our method is easy to comprehend and implement (given the fact that most of the parameters used as input are available with the traders) and gives pretty accurate results in a short span of time.In our third study, we explore MaxVaR as a risk measure in the context of options. Despite the obvious importance of MaxVaR/intra-horizon risk, it is a relatively unexplored area with only a handful of published research papers. Thus, in our study, we propose the use of MaxVaR as a risk measure in the context of options and study the differences between 10-day and 15-day VaR and MaxVaR. We use various distributions such as Pearson's type IV distribution, Johnson's SU distribution and skewed Students t distribution to model the residuals/innovations of the profit and loss (P&L) of the options in a dynamic framework so as to capture characteristics such as high excess kurtosis and skewness. We vary the moneyness of the option as well as extend our study to various other index options for robustness checks. We find that MaxVaR to VaR ratio can be as high as 1.4 at 5% confidence level for a 30-day maturity option for a 15-day risk horizon. We also compare the MaxVaR to VaR ratio for options and the underlying stock and find that the ratio is higher for the stocks than options.The final essay is developed in the context of Mixture of Distribution Hypothesis (MDH). The objective of this essay is to study the impact of volume on volatility and risk in the context of MDH. In light of the past empirical results reported by various researchers, the aim of this essay is to verify the validity of the MDH across time-periods, economies under study and distributional assumptions on the innovations. While one or more of these issues have been addressed in isolation by researchers in the past, there is little or no documented evidence of all these issues being addressed simultaneously. We also investigate the impact of volume on Value-at-Risk. To the best of our knowledge, this question has not been addressed in the context of MDH. We aim to fill this gap in the literature through our study. We find that the persistence in volatility shows negligible reduction in all the indices across sub-periods thus refuting the claims of the MDH that volume can explain the heteroscedasticity of returns. However, we do find that volume can indeed be treated as a proxy for information after the sub-prime financial crisis and volume does impact VaR as the estimates improve significantly for some of these indices which exhibit a strong correlation between volume and volatility.