Information Transmission In Joint Decision Making

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October 1997

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This paper investigates the possible equilibria in a game of information transmission with incomplete information. Two individuals are required to reach a decision between two alternatives and a decision is reached only if they can agree on it. Individuals receive private signals and can send messages to each other sequentially. However, they can only send binary messages and the sending of messages is time consuming. We investigate the possible equilibria when the individuals agree on the desired outcomes and when they do not. We show that when the players have the same preferences over outcomes there is a limited amount of disagreement that can take place. Also, that as the players become more patient the only equilibria which survive are those where information is aggregated efficiently and less delay is incurred. Further, we show that, when players are impatient an increase in impatience can serve to improve efficiency. In the case of biased players the equilibria are similar to the ones for the no-bias case. We show that the player who is more patient and who moves first is able to reach decisions favorable to himself. If both players are patient there is an equilibrium where there is a large amount of disagreement. For such a situation the imposition of a time limit on the amount of discussion allowed would improve efficiency.

I am grateful to Beth Allen, Andreas Blume, Jocelyn Evans, Robert Forsythe, James Maw, J.F. Mertens, Suresh Mutuswami, Jennifer Reinganum and John Solow for advice. All remaining errors are mine.

1. Introduction

The use of teams to organize production is ubiquitous. As such, the problem of the optimal organization of teams has, deservedly, received considerable attention from economists. This literature has largely focused on the issues arising from the provision of proper incentives for the individual members of a team to maximize joint production. Thus, team output could depend on a variable like weather and on the effort of each individual member of the team which could be unobservable. The job of the principal then is to devise compensation schemes which induce the agents to provide the proper effort (Holmstrom 1982).

While the use of teams in production is all pervading teams have other functions as well. In society certain decisions are delegated to a group of individuals who, for one reason or another are, supposedly, better qualified to take decisions on behalf of other members of society. Examples are democratically elected governments, juries and committees. One obvious reason for decision making through teams is that it mitigates the bias in decisions which might arise from allowing only one person to take decisions. Quite often another reason is that some decisions require some specific knowledge or expertise and it is felt that a group of such persons would be able to reach a better decision than any single individual acting alone. The reasoning is that these people could share their knowledge or expertise and reach superior decisions. An example is the board of directors of a firm. Thus, in summary, teams are used to mitigate individual biases and provide superior information sharing.

This issue of information transmission in teams has been relatively neglected. This statement has to be qualified by noting that team decision problems arise in many different areas of life, such as in the constitution of juries and committees, and in those particular areas such problems have received their due attention. However, we would argue that there is a information transmission aspect which is common to all team decision problems and it is the purpose of this paper to investigate this.

The problem with reaching decisions through a group of individuals is that, though they might have private information about the problem at hand, they might be reluctant to share such information. It is even possible that they might deliberately try to mislead the other members of the group about the information they have in order to achieve their preferred alternative among the menu of options available. As an example consider a union and the management bargaining over wages. Each may possess information about the state of the economy in future and could agree on the wages to be paid under some circumstances but might disagree about others. This disagreement over wages in some situations could lead them to mislead the other party about their private information. Another scenario where the same problem could crop up is in setting standards on product compatibility. This issue has been looked at in terms of a coordination game by Farrell and Saloner(1988). It is possible that the firms may agree on the standards to be set if they knew the future of the industry in some situations and disagree in others. It is also possible that they may have private information about the future of the industry which, if shared, could lead to a quick decision on standards. However, it is this which would lead to the manipulation of information revealed to achieve a desirable outcome if the state of the world were such that the two individuals preferences are opposed to each other. In contrast to standard bargaining models here the size of the pie is not known and the two, or more, parties have private information about the size. We feel that there is both a bargaining and a communication aspect to group decision making and we will try to capture this feature with our model which is similar to bargaining models.

As noted above a number of studies have been conducted in different areas which touch on the problem in question. Some of the earliest attempts to model the information sharing aspect of a group of individuals have been in the analysis of jury decision making. However, it is unlikely that there is a strong bargaining feature in juror decision making and the relationship of this literature to our present concerns is at best tangential.

Other scholars have also looked at the problem from their own perspectives. Ladha (1992) writes about the Condorcet Jury Theorem, " The theorem establishes that under certain conditions a majority of a group, with limited information about a pair of alternatives, is more likely to choose the "better" alternative than any one member of the group. The theorem, thus, provides a mathematical basis for majority rule voting and potentially an important clue to our understanding of the strength of democratic government." However, the theorem requires two assumptions : the jurors vote independently and they share a common goal. Ladha removes the assumption of independence and considers "correlated votes" because given common information and shared experiences votes are not likely to be independent. Although Ladha recognizes the lack of independence among votes the analysis conducted is not game theoretic in nature. A recent study by Austen-Smith and Banks(1995) shows that a crucial assumption in the Condorcet Jury Theorem, that people vote "sincerely", is not rational and in a Nash equilibrium it is not possible that everybody votes sincerely. Our work considers a different mechanism from voting as a way to reach decisions. It could be called decision making through consensus but we too are concerned about the efficiency of decisions reached.

The paper which comes closest to our concerns is by Sah and Stiglitz (1988). They consider three forms of organizations: committees, hierarchies and polyarchies. They evaluate the effect of organizational form on the statistical errors of project selection. In their model individuals are homogeneous in their information processing abilities but are prone to errors. Thus, given any project, individuals make their assessments on the quality of the project, i.e., good or bad. However, these decisions are subject to errors. Individuals may not approve a good project (Type-I errors) or may approve a bad project (Type-II errors). The incidence of these errors are the same across individuals. The decision making body could be constituted as a committee of n such individuals with a pre assigned majority rule. Or, the decision making body could be a hierarchy (a bureaucracy) where projects are passed on to higher levels only if they are approved at the lower levels. In contrast, a polyarchy accepts a project if any individual member does so. Using, this model and by including evaluation costs Sah and Stiglitz derive results on the relative performance of these different types of organizational modes.

We propose to take a close look at the structure of the problem of information transmission and introduce costs of delay. It is our contention that teams do not reach decisions instantaneously and so we will build a model where teams could, potentially, take time to reach a decision. We will, also, introduce different tastes so that there is possibility for disagreement. The main significance of our work is that members of the team can decide on their messages strategically. In contrast Sah and Stiglitz model messages being sent with an exogenously determined probability. Koh (1992) shows that in a hierarchy the individuals at a higher rank should take into consideration the recommendation provided by an individual at a lower rank. However, his analysis deals only with hierarchies and does not take into account strategic concerns. We would argue that the choice of signals to send should be determined by the individuals themselves after taking into consideration the parameters of the problem.

To address our concerns we build a model which mimics conversation between two individuals. This we do by constructing a game where two individuals, with private information, send messages to each other sequentially. Further, the individuals are restricted in their choice of messages they can send. Specifically they can suggest one of the two decisions under consideration. If the two individuals agree at any point in their conversation, the decision that they agree on is adopted as the decision the team reaches and the game ends. In our view individuals are naturally restricted in their ability to communicate by the nature of language and this structure highlights this view. It is also possible that this restriction is a result of an optimization problem. We could view these two individuals as two departments within an organization. Then it is quite plausible that communication between the two departments takes some predetermined form.

We investigate the Perfect Bayesian Equilibria for this game for the situations where the individuals have the same preferences and when they do not. We begin with a discussion of the general structure of the equilibria and describe our choice of out of equilibrium beliefs. For the particular out of equilibrium beliefs we choose the game ends within a limited time, which is useful for the purpose for the purpose of describing equilibria. We show, in the case of same preferences, that if the players are impatient there are equilibria which involve useless communication. At the same time there are equilibria which involve no communication at all and are efficient. The problem is that communication involves delay and is only useful if it leads to an exchange of information and better decision making in terms of lower statistical errors. Inefficient equilibria involve the sending of messages which have no information content. In such a situation increased impatience on the part of the players would be beneficial. An alternative would be to let a single individual decide. If players are patient then there is more exchange of information and, as the cost of delay is lowered further, only efficient equilibria remain.

In the case of players with different preferences the structure of the equilibria are similar to the case of same preferences. As long as one of the players is impatient the game ends within a limited time. However, there is a larger amount of disagreement and individuals who move first, or are more patient, are able to reach decisions favorable to themselves. If both individuals are patient, there is an equilibrium which involves long, though finite, discussions. In this equilibrium both individuals continue to insist on their desired outcome hoping that the other individual will concede. The game ends when one of the individuals becomes so pessimistic about the other player conceding that he concedes himself. Clearly, this equilibrium is inefficient and a limit on the amount of discussion allowed would prove helpful.

In the next section we describe our model. The general structure of the equilibria and out of equilibrium beliefs are discussed in section 3.

Sections 4 and 5 discuss the equilibria in the case of same preferences and biased preferences, respectively. Section 6 provides the conclusion.

2. Model

We will denote the state of nature by θ which can take one of two values θ_0 and θ_1. Each individual receives a signal, s, which could take one of two values s_1 or s_2 and has a prior over the state of nature, $P(\theta=\theta_0) = \xi$. Consequently, individuals can calculate the posterior $\xi(s) = P(\theta=\theta_0|s)$ using Bayes' formula and their knowledge of the conditional probability distribution between the signals and the states of nature. More generally if there are n individuals and the signals they receive were public knowledge then the common posterior will be denoted $\xi(\mathbf{s})$, where $\mathbf{s}$ will be a $(n \times 1)$ vector of s_1 and s_2. If signals were informative about the state of nature then a larger sample of signals would lead to better decision. So the emphasis will be on finding out to what extent individual information can be aggregated for joint decision making. The preferences of an individual will be given in terms of costs of Type-I and Type-II errors. Thus if we denote the decision $\theta=\theta_0$ as d_0 and $\theta=\theta_1$ as d_1 the costs of making the wrong decision are shown below.

	$\theta=\theta_0$	$\theta=\theta_1$
d_0		a_1
d_1	b_1	

(1)

From decision theory an individual would choose to minimize the costs of reaching a decision and would use the rule d_0 if $\xi(s) > \frac{a_1}{a_1 + b_1}$, d_1 if $\xi(s) < \frac{a_1}{a_1 + b_1}$, d_0 or d_1 if $\xi(s) = \frac{a_1}{a_1 + b_1}$. In the games we will look at the individuals will be restricted to binary messages, m, which can be d_0 or d_1, in any one stage. At first sight, this might seem too restrictive. However, it is plausible that the message space is smaller than the number of possible signals that we may receive and this restriction merely serves to highlight this possibility. The restriction of messages to these two possibilities could, also, be a decision which has been separately undertaken to conserve costs. The outcome of these games will also be d_0 or d_1. Thus, we could be talking about the decision theory of teams. We will begin by looking at a two person game.

We will call these two individuals 1 and 2 with costs a_1, b_1 for 1 and a_2, b_2, for 2. Both of them receive a signal $s \in (s_1, s_2)$ and have a prior ξ over the states of nature. The probability with which any individual receives the signal s_1 is π. The posteriors 1 and 2 would form, if they knew the signals 1 and 2 have received are $\xi(s_1,s_1)$, $\xi(s_1,s_2)$ or $\xi(s_2,s_2)$. For ease of notation we will call these π_{11}, π_{12} and π_{22}. Both 1 and 2 know the values of these posteriors. The game proceeds by 1 sending a message $m_1 \in \{d_0, d_1\}$ in stage 0. If 2 sends the same message ($m_1 = m_2$) in stage 1 then the game ends and payoffs are received. If 2 does not agree with 1 then both players incur a cost of delay by a factor $D(D > 1)$ and the game continues with 1 sending a message. If 1 agrees with 2's message then the game ends, otherwise the game continues with a further cost of delay. The structure is quite similar to bargaining games and comes under the general classification of multistage games with observed actions. This structure is given exogenously and might not be appropriate in some circumstances. For example the players could restart discussions even if they do agree on a particular verdict. Thus we should also consider the question whether this structure is optimal for reaching decisions. We would argue that this is good starting point. The payoffs are in terms of expected costs. For example, the game might end in the first stage with 1 saying d_0 and 2 agreeing. Then 1's payoff will be the cost of Type-I error $a_1P(\theta=\theta_1|s_1,m_2)$. If 1 receives a signal s_1 and infers from m_2 that 2's signal is s_1 with probability ϕ, then 1's payoff is $a_1[\ \phi(1 - \pi_{11}) + (1 - \phi\ )(1 - \pi_{12})]$. The structure of the game is shown in Figure 1.

We will define $m_i^k \in \{d_0,d_1\}$ as the message player i sends in stage k. Then the history h^{k+1} will be defined as $\{m_1^o,m_2^1,\ldots\ldots,m_i^k\}$. Notice that the game starts in stage 0 with 1 sending the message m_1^o and that 1 and 2 take turns sending messages. The game ends if $m_i^{k-1} = m_j^k$. A behavior strategy σ_i is the probability $\sigma_i^k(m_i^k|h^k,s_t)$

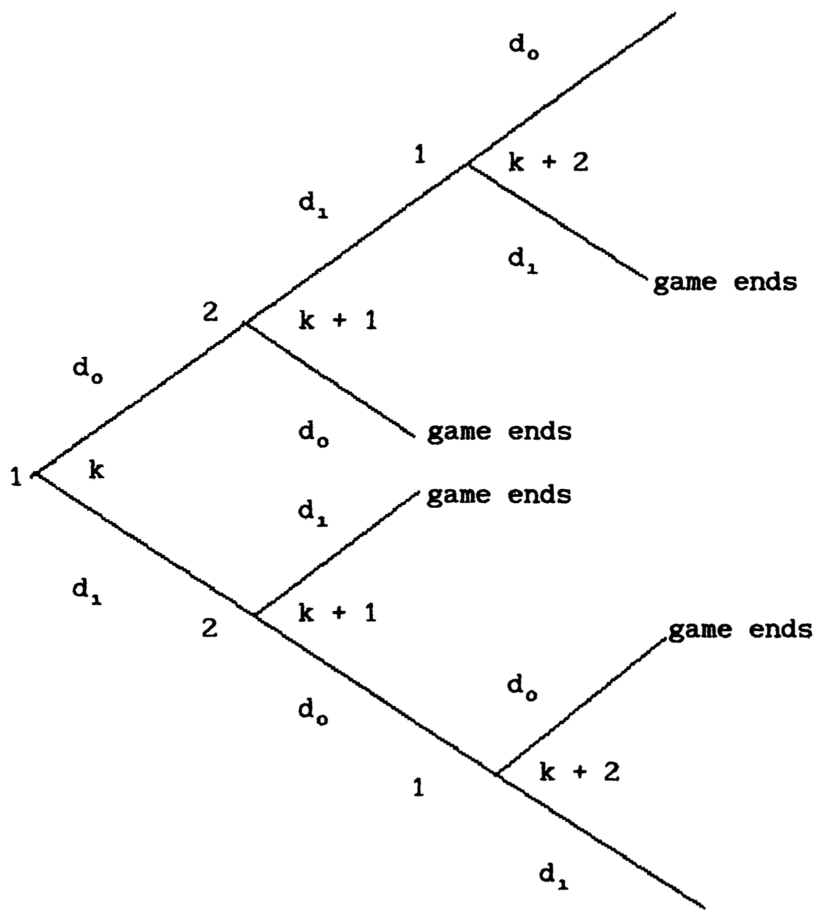

Figure 1. Game tree

that player i sends message m_i^k at stage k when he has observed the signal s_t

and history h^k. We will define $\mu_i^k(s_t|h^k)$ as the belief player i has about player j's type in stage k, given the history h^k. We will require that these beliefs are updated using Bayes' rule whenever possible. Thus

$$\mu_i^{k+1}(s_t|h^k,m_j^k) = \frac{\sigma_j^k(m_j^k|h^k,s_t)\mu_i(s_t|h^k)}{\sum_{t=1}^{2}\sigma_j^k(m_j^k|h^k,s_t)\mu_i(s_t|h^k)}. \tag{2}$$

Let us denote $\sigma_i = \left\{\sigma_i^k\right\}_{k=0}^{\infty}$; and $\lambda^{0k}(\sigma_i,\sigma_j)$ $(\lambda^{1l}(\sigma_i,\sigma_j))$ as the probability that the game ends in stage k(l) with a decision of $d_0(d_1)$. Then player i's payoff $u_i(\sigma_i|s_t)$ is given by

$$\sum_{k=1}^{\infty}\sum_{t=1}^{2}\lambda^{0k}(\sigma_i,\sigma_j)D^{k-1}a_i(1-\pi_{it})\mu^0(s_t) + \sum_{l=1}^{\infty}\sum_{t=1}^{2}\lambda^{1l}(\sigma_i,\sigma_j)D^{l-1}b_i\pi_{it}\mu^0(s_t) \tag{3}$$

where $\mu^0(s_t)$, the probability of player j receiving the signal s_t, is π or $1-\pi$. Let σ define the strategy profile $\{\sigma_i,\sigma_j\}$. Then a perfect Bayesian equilibrium is a (σ,μ) such that

$$u_i(\sigma|h^k,s_r,\mu_i^k(s_t|h^k)) \geq u_i((\sigma_i',\sigma_j)|h^k,s_r,\mu_i^k(s_t|h^k)) \;\forall\, k, \forall\, \sigma_i'. \tag{4}$$

By Fudenberg and Tirole(1991) this equilibrium will also be a sequential equilibrium as defined by Kreps and Wilson(1982).

3. Preferences, Beliefs and Sequential Equilibria

In this section we will discuss the structure of preferences we will be investigating in Section 4 of the paper. We will also discuss the structure of sequential equilibria and the structure of out of equilibrium beliefs for these preferences. As a part of the definition of a sequential equilibrium it is necessary to outline strategies for all possible histories including those that would not be reached if the players played their equilibrium strategies. This imposes a rather large notational burden which we will eliminate by showing the general structure of all sequential equilibrium and going on to discuss specific equilibria with much simpler notation.

We will start by looking at the situation where the two individuals have the same preferences. In a number of situations which involve the use of teams to reach decisions it would be appropriate to model team members with identical preferences. There is the notion that if individuals in a team have similar goals then it is easier to reach better decisions. An investigation of this notion requires a study of decision making with identical preferences. This notion is captured by the assumption that $a_i = a$, $b_i = b$ for i = 1, 2. We will further assume that

$$\pi_{11} > \pi_{12} > \frac{a}{a + b} > \pi_{22}. \tag{5}$$

From the way posteriors have been defined it is clear that the appropriate decisions for the events (s_1,s_1), (s_1,s_2) and (s_2,s_2) are d_0, d_0 and d_1 respectively. Under these circumstances there is a need for transmission of information across individuals. A situation where the correct decision is d_0 in all events would not be interesting. We could have investigated the situation where $\pi_{11} > \frac{a}{a + b} > \pi_{12} > \pi_{22}$, but this would be symmetric to (5) and the types of equilibria would be similar.

In games of incomplete information the structure of equilibria depend crucially on the out of equilibrium beliefs. Here we will argue that if both types of 1 are supposed to say d_0 in equilibrium then if d_1 is observed 2 will believe that 1 is type s_2. Similarly if both types of 2 are supposed to agree with d_0 and d_1 is instead observed then 1 will believe that 2 is type s_2. The reverse holds if the signals are supposed to be d_1 for 1 and 2 is supposed to agree with that message. From (5) we can see that type s_1 would always settle for d_0 if he were left to decide for himself and type s_2 could say d_1. Thus, these beliefs are plausible. We will, later, discuss these beliefs from the point of the criteria imposed by the literature on out of equilibrium restrictions. There we will show that these beliefs satisfy the Cho and Kreps (1987) intuitive criterion. First we will show that in any equilibrium the game should last only four stages.

Proposition 3.1: Let K be the last stage where there is a disagreement, then $K \leq 3$.

Proof: The proof will be through contradiction. Suppose $K = 4$, then there are two possible sequences of messages which could occur: (i) $\{d_0,d_1,d_0,d_1,d_0,d_0\}$ or (ii) $\{d_1,d_0,d_1,d_0,d_1,d_1\}$. In (i) 1 sends the message d_0 to which 2 replies with d_1 in the first stage, this is repeated in the second and third stage, and in the fourth stage 1 repeats the message d_0 to which 2 agrees. (ii) can be interpreted in a similar manner. Consider (i): when 2 says d_1 in the first stage 2 must have received s_2 as a signal. Consequently when 1 repeats his message d_0 1 must have received a signal s_1 and so the optimal decision is d_0 and 2 should agree with 1's message, d_0. Similarly in (ii) when 2 says d_0 to 1's d_1, $1(s_1)$ should agree in the next stage. If d_1 is observed player 1 must be type s_2. In stage 3, $2(s_2)$ would agree and end the game. If that does not happen player 2 must be type s_1 and player 1 should agree in the next stage. Thus $K \leq 3$.

This result derives from the limited amount of information that needs to be transmitted and the preferences of the two individuals. Type s_1 of either player has a dominant strategy in agreeing with d_0 and this serves to differentiate between types. Once all the information gets revealed there can be no disagreement since the two players have identical preferences.

The question still remains as to how the game proceeds if for some reason it has not ended by the fourth stage. We will argue that if the game has not ended type s_2 would immediately agree in the next stage with the previous message. Type s_1 will either say d_0 in every stage or will agree with the message in the last stage.

Proposition 3.2:

For i = 1,2, (i) $\sigma_i(d_0|h^k,s_1) = 1\ \forall\ k \geq 3$, or, $\sigma_i(d_0|h^k,s_1) = 1$ if $h^k = \{h^{k-1},d_0\}$, $\sigma_i(d_0|h^k,s_1) = 0$ if $h^k = \{h^{k-1},d_1\}$, $\forall\ k \geq 3$, and (ii) $\sigma_i(d_0|h^k,s_2) = 1$ if $h^k = \{h^{k-1},d_0\}$, $\sigma_i(d_0|h^k,s_2) = 0$ if $h^k = \{h^{k-1},d_1\}$, $\forall\ k \geq 3$.

Proof: There are two possible histories: (i) $h^k = \{h^{k-1}, d_0\}$ and (ii) $h^k = \{h^{k-1}, d_1\}$. In (i) player i must have said d_1 to which player j has responded with d_0. By player i's beliefs, in equilibrium or out of equilibrium, player j must be type s_1. Then player i should say d_0 and end the game. For the second history player j is type s_2 and if player i is also type s_2 he should end the game by saying d_1. If he is type s_1 then if D is low enough he would say d_0; otherwise he should say d_1 and end the game.

The discussion of equilibria in Section 4 will concentrate on the strategies of the players in the first four stages only by force of Proposition 3.1 and 3.2. The rest of this section will involve a detailed discussion of beliefs and sequential equilibria. We will show that there are other beliefs which satisfy the Cho and Kreps criteria but that for these the equilibrium outcomes are the same as with our choice of beliefs. The reader can without loss of continuity go on to Section 4.

To investigate the justification for these particular beliefs we will look at a couple of equilibria. It should be intuitively clear that given a sufficiently high cost of delay and a high probability of the event (s_1,s_1) there should be an equilibrium where both types of player 1 say d_0 and both types of player 2 agree with player 1's message. The question is how high should the values of π and D be to support such an outcome. Consider the following equilibrium

$\sigma_1^k(d_0|h^k,s_t) = 1$ for $h^k = h^0$ and $h^k = \{h^{k-1},d_0\}$, $t = 1,2.$

$\sigma_1^k(d_0|h^k,s_t) = 0$ for $h^k = \{h^{k-1},d_1\}$, $t = 1,2.$

$\sigma_2^k(d_0|h^k,s_t) = 1$ for $h^k = \{h^{k-1},d_0\}$, $t = 1,2.$

$\sigma_2^k(d_0|h^k,s_t) = 0$ for $h^k = \{h^{k-1},d_1\}$, $t = 1,2.$

with out of equilibrium beliefs

$\mu(s_1|h^k) = 1$ for $h^k = \{h^{k-1},d_0\}$

$\mu(s_1|h^k) = 0$ for $h^k = \{h^{k-1},d_1\}$.

In this equilibrium both types of player 1 say d_0 in the first stage and both types of player 2 agree with player 1's message in the second stage. If player 1 says d_1 player 2's out of equilibrium belief requires him to think he is facing type s_2 and he, consequently, agrees with 1's message d_1. Also, if player 1 says d_0 and player 2 says d_1, then player 1 believes that he is facing type s_2 and in the next stage agrees with 2's message and says d_1 to end the game. Note that if the game has not ended there are two possible histories which can hold; both of which require player 1 and 2 to alternate between d_0 and d_1 with the difference being that player 1 starts the game by saying d_0 or d_1. At any history both types of both players are required in the equilibrium to agree with the previous message and depending on whether they disagree by saying d_0 or d_1 are deemed to be type s_1 and s_2 respectively.

To find out the values of the parameters D and π for which this equilibrium will hold first consider $2(s_1)$. If 1 says d_1 then given player 2's belief the payoff for agreeing with 1 is $b\pi_{12}$ while that from saying d_0 is $Da(1 - \pi_{12})$. Player 1 would agree with player 2's second stage message d_0 according to the equilibrium and since there would be a delay of one stage the payoff has to be multiplied by D. So the first condition we would require is

$$b\pi_{12} \le Da(1 - \pi_{12}) \text{ or } D \ge \tilde{D} \equiv \frac{b\pi_{12}}{a(1-\pi_{12})}. \tag{6}$$

Although the ideal decision for the event (s_1,s_2) is d_0 it is worthwhile to reach a decision d_1 rather than incur delay to reach a decision d_0. $1(s_1)$ knows that the possible events are (s_1, s_1) and (s_1,s_2); both of which merit the decision d_0 and since both types of s_2 are going to agree with player 1's message player 1 type s_1 says d_0. Player $1(s_2)$ has to weigh the payoffs from reaching a decision d_0 or d_1. Sending the message d_0 produces a payoff of

$$a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi),$$

the probability of the events (s_1,s_2) and (s_2,s_2) and the losses associated with d_0 while that from sending the message d_1 obtains

$$b\pi_{12}\pi + b\pi_{22}(1 - \pi)$$

Thus it would be optimal to say d_0 if

$$a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi) \le b\pi_{12}\pi + b\pi_{22}(1 - \pi)$$

$$\text{or, } \pi \geq \pi_1 \equiv \frac{\frac{a}{a+b} - \pi_{22}}{\pi_{12} - \pi_{22}}. \tag{7}$$

The last requirement on π is derived from $2(s_2)$'s decision to agree with player 1 when he says d_0. The payoff from this action gives a payoff of

$a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi)$,

while that from saying d_1 would provide a payoff of

$D\{b\pi_{12}\pi + b\pi_{22}(1 - \pi)\}$.

Thus, $2(s_2)$ will say d_0 if

$$a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi) \leq D\{b\pi_{12}\pi + b\pi_{22}(1 - \pi)\} \tag{8}$$

$$\text{or, } \pi \geq \pi_2 \equiv \frac{\frac{a}{a+Db} - \pi_{22}}{\pi_{12} - \pi_{22}}. \tag{9}$$

$2(s_1)$ saying d_0 when player 1 says d_0 is a dominant strategy and so is saying d_1 when player 1 says d_1 given player 2's out of equilibrium beliefs. The reader can check that the strategies enumerated in the equilibrium for the histories beyond the first two stages are optimal given the beliefs and the values of the parameters.

We can envisage a different sequential equilibrium which produces the same outcome as the above equilibrium but for a different set of parameters. Consider the equilibrium

$\sigma_1^k(d_0|h^k,s_t) = 1$ for $h^k = h^0$ and $h^k = \{h^{k-1},d_0\}$, $t = 1,2$.

$\sigma_1^k(d_0|h^k,s_t) = 1$, $t = 1$, $h^2 = \{d_0,d_1\}$; $\sigma_1^k(d_0|h^k,s_t) = 0$, $t = 2$, $h^2 = \{d_0,d_1\}$

$\sigma_1^k(d_0|h^k,s_t) = 0$, for $h^k = \{h^{k-1},d_1\}$, $t = 1,2$, $h^2 \neq \{d_0,d_1\}$.

$\sigma_2^k(d_0|h^k,s_t) = 1$ for $h^k = \{h^{k-1},d_0\}$, $t = 1,2$.

$\sigma_2^k(d_0|h^k,s_t) = 0$ for $h^k = \{h^{k-1},d_1\}$, $t = 1,2$.

with out of equilibrium beliefs

$\mu(s_1|h^k) = 1$ for $h^k = \{h^{k-1}, d_0\}$

$\mu(s_1|h^k) = 0$ for $h^k = \{h^{k-1}, d_1\}$, $h^2 \neq \{d_0, d_1\}$.

$\mu(s_1|h^k) = 1$ for $h^2 \neq \{d_0, d_1\}$.

The structure of this equilibrium is the same as the previous one except that for the history $\{d_0, d_1\}$, $1(s_1)$ says d_0 since he believes that it is type s_1 who has sent the out of equilibrium message d_1. For this to be optimal it is necessary that

$$Da(1 - \pi_{11}) \leq b\pi_{11}. \tag{10}$$

As before $1(s_1)$ will say d_0 in the first stage and so will $1(s_2)$ if (7) holds. Similarly $2(s_1)$ will agree with 1's message d_0. The conditions required for $2(s_2)$ to agree with d_0 will be different. The payoff from agreeing are the same as before

$$a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi),$$

while that from saying d_1 is

$$D^2a(1 - \pi_{12})\pi + Db\pi_{22}(1 - \pi).$$

Thus saying d_0 is optimal if

$$a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi) \leq D^2a(1 - \pi_{12})\pi + Db\pi_{22}(1 - \pi), \tag{11}$$

$$\text{or, } \pi \geq \pi_8 \equiv \frac{\dfrac{a}{a + Db} - \pi_{22}}{\dfrac{(D^2 - 1)a(1 - \pi_{12})}{a + Db} + \dfrac{a}{a + Db} - \pi_{22}} \tag{12}$$

We will argue that this equilibrium is not plausible. Intuitively, this equilibrium allows the decision d_0 to be reached with a lower value of D and the way this is achieved is by $1(s_1)$ making a threat that in case 2 does send the out of equilibrium message d_1 $1(s_1)$ will believe that 2 is type s_1 and will say d_0. However, the question is whether 1 could entertain this belief when faced with this out of equilibrium message. In the previous equilibrium we are driven by condition (8) whereas here we have to contend with (11). Denote the values of D for which (8) and (11) hold with equality as D_1 and D_2 respectively. Note that the left hand side of (8) and (11) are the same while the right hand side of (8) is less than (11), since $D \geq \tilde{D}$ is a condition that must hold in this equilibrium as well. Consequently, for a given value of π, $D_1 > D_2$. Thus, there would be a value of D, $D_1 > D > D_2$, for which (8) would not hold but (11) would. For such a value of D if $1(s_1)$ were to believe that player 2 is type s_2 when he sends the out of equilibrium message d_1 and were to choose his best response for that belief then $2(s_2)$ would deviate from d_0. This argument is similar to Cho and Kreps(1987) intuitive criterion and would therefore agree with divinity and universal divinity criteria of Banks and Sobel(1987).

Of course for values of D for which (8) holds would imply that (11) holds as well. Thus, for such values of D it would not be possible to rule out the second equilibrium. Our interest in various equilibria are derived from the constraints on D and π they allow us to impose for different outcomes. The second equilibrium gives the same outcome as the first for values lower than D_1 but then such an equilibrium would not satisfy the Cho and Kreps criterion. For values of $D \geq D_1$ the first and the second equilibrium will provide the same outcomes. Therefore we will neglect the second equilibrium and the associated out of equilibrium belief for the rest of the analysis. In a similar manner one could generate an equilibrium where player 2 would believe that player 1 is type s_1 if he says d_1 and therefore say d_0. This would make $1(s_2)$ more inclined to say d_0, but there would be a value of D such that $1(s_2)$ would be willing to deviate if such deviation could bring about a different response from $2(s_1)$. For $D \geq \tilde{D}$, it is not possible to restrict beliefs any further because the intuitive criterion and divinity restrict themselves to signaling games. Grossmann and Perry(1986) consider infinite horizon games but their concept of perfect sequential equilibrium is not helpful in this case. The concept of stability due to Kohlberg and Mertens(1986) does not help either since it deals with finite horizon games.

This does not raise any problems because the beliefs and consequently the actions taken at stages beyond the first four stages do not affect the values of π and D for which the desired outcomes occur. Consider the first equilibrium: we stipulated that beyond the second stage if player i has sent the message d_0 in the previous stage and player j says d_1 then player i believes j is type s_2. The opposite will be true if player i says d_1 and player j says d_0. We showed the sequential equilibrium for such beliefs. Suppose instead of these beliefs player 2 faced with the history $\{d_0,d_1,d_0\}$ believes he is facing s_2. For the appropriate value of D, $2(s_2)$ would say d_1 and we would have a different sequential equilibrium but that would not change the value of π and D where we have the equilibrium outcome that both types say d_0 and both types agree to this message. The point is the stipulation of beliefs after the game has ended do no affect the equilibrium outcome.

The restrictions on beliefs that we have stipulated allow us to get rid of some equilibria entirely. For example there is a sequential equilibrium where both types of 1 send the message d_1 and both types of player 2 agree with this decision. To sustain this equilibrium we need the out of equilibrium belief that if 1 sends the out of equilibrium message d_0, then player $2(s_2)$ would believe that player 1 is type s_2 and D would be small enough to warrant insisting on d_1. However, we can show that this equilibrium does not satisfy the Cho and Kreps criterion.[1]

4. Same Preferences

The previous section detailed the structure of the sequential equilibria beyond the first three stages. In this section we will therefore concentrate on the first three stages. To this purpose we will further divide our investigations into two parts. First we will consider the case when $D \geq \tilde{D}$. In our earlier discussion we noted that this condition implies that it is not worthwhile to reach a decision of d_0 in the event (s_1, s_2) when the posterior is π_{12}. The condition $D \geq \tilde{D}$ can be rewritten as $\pi_{12} \leq \frac{Da}{Da + b}$. The values of the posteriors and the various parameters are shown in Figure 2. Given this constraint on the cost of delay we can put some additional structure on the equilibria. This would also help us in studying individual equilibria.

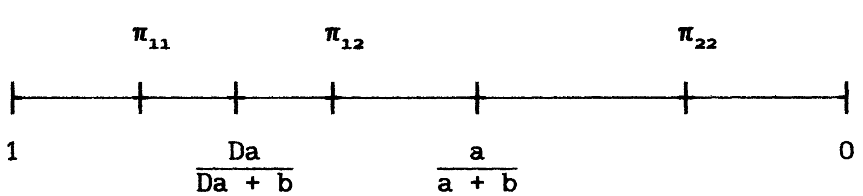

Figure 2

Proposition 4.1: If $D \geq \tilde{D}$, $K \leq 2$.

Proof: The above statement says that for any possible history at the end of the first two stages player 1 of both types would end the game by agreeing with the last message. There are two possible histories to consider: (i) $\{d_0, d_1\}$ and (ii) $\{d_1, d_0\}$. In (i) only type s_2 could disagree with d_0 and so if player 1 is type s_1 he would realize that the event is (s_1,s_2) with a posterior of π_{12} in which case it is not worthwhile to say d_0. Note that $2(s_1)$ would always agree with d_0; it could happen that both players are supposed to agree but d_1 is observed instead but then the out of equilibrium belief will be that player 2 is type s_2. If 1 is type s_2 the correct decision is d_1 so it is optimal to agree. In (ii) if player 2 says d_0 in the second stage he must be of type s_1. When player 1 says d_1 in the first stage he could be either of the two types. If he is of type s_1 and player 2 is type s_2 then since $D \geq \tilde{D}$ player 2 would agree with d_1. If player 1 was type s_2 as was player 2, d_1 would be the correct decision. So agreeing with d_1 is a dominant strategy for $2(s_2)$. Thus player 1 would agree with d_0 in the second stage.

Armed with proposition 4.1 we go on to investigate the possible equilibria. Since the game ends within two stages there could be two possibilities; (i) equilibria in which the game ends in one stage and (ii) where the game could take two stages. We will look at equilibria of the first kind in the next proposition.

Proposition 4.2: For $D \geq \tilde{D}$ there are two one stage equilibrium where player 2 agrees with player 1: (i) both types of player 1 say d_0 for $\pi \geq \{\pi_1,\pi_2\}$, (ii) $1(s_1)$ says d_0 and $1(s_2)$ say d_1 for $\pi \leq \pi_1$, where

$$\pi_1 = \frac{\frac{a}{a+b} - \pi_{22}}{\pi_{12} - \pi_{22}} \text{ and } \pi_2 = \frac{\frac{a}{a+Db} - \pi_{22}}{\pi_{12} - \pi_{22}}.$$

Proof: See Theorem 4.1 in appendix.

In equilibrium (i) player $1(s_1)$ would always say d_0 since player 2 is going to agree. From Figure 2 player 1 knows that he is either at π_{11} or π_{12} and that for both of these situations d_0 is the correct decision. Player $1(s_2)$ has to weigh his options: either way he cannot avoid the possibility of reaching a wrong decision. From figure 2 he is either at the point π_{12} or at π_{22}. The term in the numerator of π_1, $\frac{a}{a+b} - \pi_{22}$, can be interpreted as the loss from saying d_0, while that in the denominator is the sum of the terms $\pi_{12} - \frac{a}{a+b}$ and $\frac{a}{a+b} - \pi_{22}$, i.e., $\pi_{12} - \pi_{22}$. Thus π_1 can be interpreted as the ratio of the loss from saying d_0 to total losses. Thus if $\pi \geq \pi_1$ it is cheaper to say d_0.

The situation is similar for player 2. $2(s_1)$ agrees with d_0 since it is the correct decision. $2(s_2)$ faces the problem that both types of player 1 say d_0, so that player 1's message is not informative, and if he disagrees and says d_1 both types of player 1 would agree and end the game. Then player 2 says d_0 if $\pi \geq \pi_2$ which is similar in structure to π_1 and can be interpreted in the same way. The added feature is that disagreeing with 1 imposes delay which shows up in π_2.

From a decision theoretic perspective the problem the team faces is two-fold. First, given the high value of D communication is costly; second, in the absence of communication it is not possible to avoid the possibility of reaching the wrong decision in some cases. In case of the event (s_2,s_2) the proper action is d_1, and the two players could, for example, reach this decision if when player 1 says d_0, $2(s_2)$ could reveal his information by sending the message d_1. However, he is not willing to undertake this action because it would impose delay. Thus the team as a whole only uses the information available to player 1. This could be interpreted as a version of the optimal stopping rule.

In equilibrium (ii) player 1 separates with type s_1 saying d_0 and type s_2 saying d_1. Then player 2 knows what message player 1 has received when it is his turn to move. Player $2(s_2)$ agrees with player 1's message, d_0 or d_1, because in each case the right decision is being reached. $2(s_1)$ obviously agrees with d_0 and would have contemplated disagreeing with d_1 were not the cost of delay been high.

We can see from a comparison of π_1 and π_2 that the former is always greater than the latter. Thus the interval [0 1] is divided into two regions, $\pi \geq \pi_1$ and $\pi < \pi_1$. This leads to our next proposition.

Proposition 4.3: For $D \geq \tilde{D}$ a one stage equilibrium always exists.

Corollary: For $D \geq \tilde{D}$ a pure strategy Nash equilibrium always exists.

We now turn our attention to equilibria which could require two stages. Proposition 4.4 describes these.

Proposition 4.4: There are two two-stage equilibrium. They are

(i) 1 of both types say d_0, $2(s_2)$ disagrees for $\pi_2 \geq \pi \geq \max\{\pi_3, \pi_4\}$.

(ii) $1(s_1)$ says d_1, $1(s_2)$ says d_0; $2(s_1)$ disagrees on d_1, $2(s_2)$ disagrees on d_0 for $\pi_5 \geq \pi \geq \pi_6$, $D \leq \min\{ \frac{a(1 - \pi_{22})}{b\pi_{22}}, \frac{b\pi_{11}}{a(1 - \pi_{11})}\}$, where

$$\pi_3 \equiv \frac{\frac{(D - 1)b\pi_{12}}{a + b}}{\frac{(D - 1)b\pi_{12}}{a + b} + \pi_{11} - \frac{a}{a + b}}, \quad \pi_4 \equiv \frac{\frac{(D - 1)b\pi_{22}}{a + b}}{\frac{(D - 1)b\pi_{22}}{a + b} + \pi_{12} - \frac{a}{a + b}}$$

$$\pi_5 \equiv \frac{b\pi_{12}}{a(1 - \pi_{11}) + b\pi_{12}}, \quad \pi_6 = \frac{b\pi_{22}}{a(1 - \pi_{12}) + b\pi_{22}}.$$

Proof: See Theorem 4.1 in appendix.

In equilibrium (i) player $2(s_2)$ disagrees with d_0 and from our discussion of Proposition 4.2 the required condition is $\pi \leq \pi_2$. Both types of player 1 are going to say d_0 even though they may be forced to change their mind by player $2(s_2)$. The necessary requirements on π for $1(s_1)$ and $1(s_2)$ are $\pi \geq \pi_3$ and $\pi \geq \pi_4$ respectively. Like before we can interpret these conditions as the ratio of loss from saying d_0 to total losses. The terms in the numerator show the loss from delay while the denominator includes the loss from taking the wrong decision. Even though in the event (s_1,s_2) the wrong decision would be taken the loss from this does not enter into π_3 because it is not possible to avoid this cost.

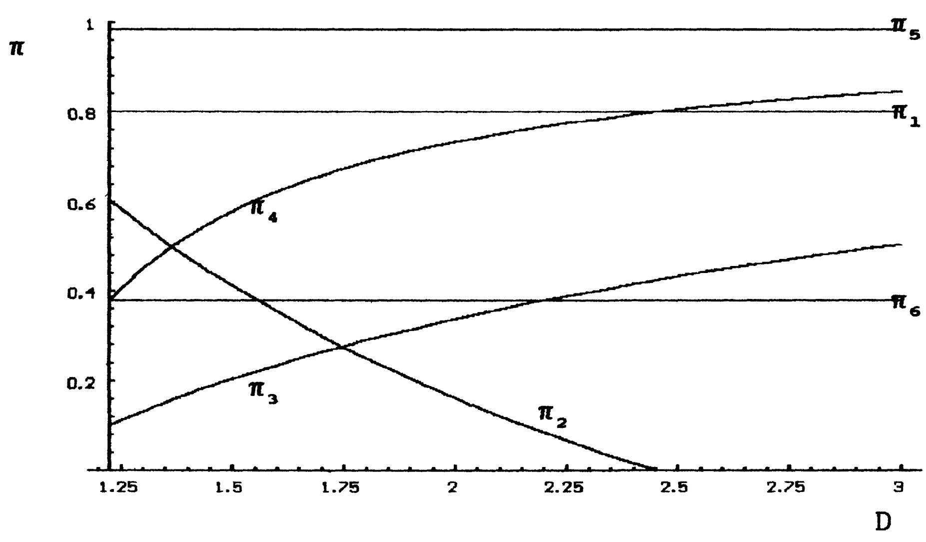

Figure 3. $\pi_{11} = 0.99$, $\pi_{12} = 0.5$, $\pi_{22} = 0.25$, $a = 45$, $b = 55$, $D \geq 1.25$.

In (ii) player 1 separates though in, what might seem, a perverse manner. Player $1(s_1)$ says d_1 and $1(s_2)$ says d_0. Player 2 is thus aware of the signals received by player 1 and know the result of agreeing with these messages. Consider $2(s_1)$, if player 1 says d_0 then the event which has occurred is (s_1,s_2) and d_0 is the correct decision. If player 1 says d_1 then the event which has occurred is (s_1,s_1) and the correct decision is d_0. The question is whether D is low enough to warrant disagreement and hence the correct decision. $2(s_2)$ faces the same dilemma when faced with the message d_0. In (ii) D is low enough for both $2(s_1)$ and $2(s_2)$ to disagree. These equilibria are shown in Figure 3.

A comparison of (i) and (ii) in proposition 4.2 shows that they are equivalent in terms of the cost of losses from decisions. The loss from (i) is

$$\pi^2 a(1-\pi_{11}) + 2\pi(1-\pi)a(1-\pi_{12}) + (1-\pi)^2 a(1-\pi_{22}). \tag{13}$$

This is the loss from the particular decisions reached in the various events and will be called the information cost. The first term shows the cost from deciding on d_0 in the event (s_1,s_1), $a(1-\pi_{11})$ multiplied by the probability of 1 and 2 receiving the signal s_1, π^2. The remaining terms are derived in a similar manner and there is no cost of delay. Similarly the cost in (ii) is

$$\pi^2 a(1-\pi_{11}) + \pi(1-\pi)a(1-\pi_{12}) + \pi(1-\pi)b\pi_{12} + (1-\pi)^2 b\pi_{22}. \tag{14}$$

A comparison of (13) and (14) shows that the loss from (i) is less than (ii) if $\pi \geq \pi_1$. Thus while neither (I) nor (II) are perfect from the point of reaching optimal decisions, (i) decides on d_0 in the event (s_2,s_2) while (ii) sometimes decides on d_1 in the event (s_1,s_2), both are equivalent in the amount of loss inflicted. The loss from (i) in proposition 4.4 is

$$\pi^2 a(1-\pi_{11}) + \pi(1-\pi)a(1-\pi_{12}) + D\pi(1-\pi)b\pi_{12} + D(1-\pi)^2 b\pi_{22}$$

which can be written as

$$\pi^2 a(1-\pi_{11}) + \pi(1-\pi)a(1-\pi_{12}) + \pi(1-\pi)b\pi_{12}$$
$$+ (1-\pi)^2 b\pi_{22} + (D-1)\pi(1-\pi)b\pi_{12} + (D-1)(1-\pi)^2 b\pi_{22}. \tag{15}$$

The last two terms show the delay cost and it is clear from a comparison of (14) and (15) that this equilibrium is not as efficient as (ii) in proposition 4.2. The cost from (ii) in 4.4 is

$$D\pi^2 a(1-\pi_{11}) + \pi(1-\pi)a(1-\pi_{12}) + \pi(1-\pi)b\pi_{12} + D(1-\pi)^2 b\pi_{22} \tag{16}$$

and we can seen that the information cost is the same as that in (15) but the delay cost is $(D-1)\pi^2 a(1-\pi_{11}) + (D-1)(1-\pi)^2 b\pi_{22}$. Thus (i) and (ii) in 4.2 are the most efficient equilibria which is restated in proposition 4.6.

Proposition 4.6: For $D \geq \tilde{D}$ one stage equilibria are more efficient than two stage equilibria.

If $D \geq \dfrac{b\pi_{22}}{a(1 - \pi_{22})}$ then equilibria (i) and (ii) in 4.4 do not exist. This condition violates the condition required for (ii) and, as can be observed from Figure 2, π_2 becomes negative and so the condition $\pi \leq \pi_2$, necessary for (i), can no longer be satisfied. So for fairly high values of D the equilibrium is going to be efficient. This would suggest that if $D \geq \tilde{D}$, a further increase in D could be beneficial though it should also be suggested that this is a circuitous route to efficiency and as good decisions would be reached by letting just one individual decide. This seems to a blow to team decision making and we investigate in the next propositions whether things get any better when D is lower.

Proposition 4.7: For $D \leq \tilde{D}$ there is a one stage equilibrium where player 2 agrees with player 1 and both types of player 1 say d_0 which exists if $\pi \geq \max\{\pi_7, \pi_8\}$ where

$$\pi_7 \equiv \frac{\dfrac{a}{a + b} - \pi_{22}}{\dfrac{(D^2 - 1)a(1 - \pi_{12})}{a + b} + \dfrac{a}{a + b} - \pi_{22}},$$

$$\pi_8 \equiv \frac{\dfrac{a}{a + Db} - \pi_{22}}{\dfrac{(D^2 - 1)a(1 - \pi_{12})}{a + Db} + \dfrac{a}{a + Db} - \pi_{22}}$$

Proof: See Theorem 4.2 in appendix.

This equilibrium is similar to the first equilibrium in Proposition 4.2. The conditions for its existence are however different. Since $D \le \tilde{D}$ and player 2 agrees with 1, player $1(s_1)$ will always say d_0 and player $2(s_1)$ will agree with this decision. Player $1(s_2)$ has to choose between d_0 and d_1. If he says d_0 and player 2 is type s_2 the wrong decision will have been reached with a relative loss of $\frac{a}{a + b} - \pi_{22}$. If on the other hand he says d_1 player $2(s_1)$ will conclude that he is type s_2 and given the low value of D insist on d_0. $2(s_2)$ would agree with d_1 and it would be the correct decision. Thus saying d_1 would lead to the correct decision in any case but possibly with delay. The first term in the denominator shows this cost. Thus, like before, π_7 can be interpreted as the ratio of the cost of saying d_0 to the total cost.

Player $2(s_2)$ faces a similar problem. If he says d_1 when 1 says d_0 it is possible that player 1 is type s_2 and would agree. Then the correct decision would have been reached but with delay. If 1 is type s_1 then he would disagree and consequently there would have been two rounds of disagreement and that would account for the power on the delay term in the denominator. The numerator shows the cost of saying d_0, $\frac{a}{a + Db} - \pi_{22}$, and this is less than the corresponding term for $1(s_2)$, $\frac{a}{a + b} - \pi_{22}$. The cost of delay implies that it is cheaper for $2(s_2)$ to agree with d_0 than it is for $1(s_2)$ to say d_0. Even though the players have the same preferences the structure of the game and the cost of delay induces different costs of reaching a decision on the two possible outcomes.

Proposition 4.8: For $D \le \tilde{D}$ there are two two-stage equilibria:

(i) $1(s_1)$ says d_0 and $1(s_2)$ says d_1, $2(s_1)$ disagrees and in next stage $1(s_2)$

agrees for $\pi \leq \pi_7$.

(ii) $1(s_1)$ says d_1 and $1(s_2)$ says d_0, $1(s_1)$ disagrees with d_1, $2(s_2)$ disagrees with both d_0 and d_1 for $\pi_5 \geq \pi$, $D \leq \dfrac{a(1-\pi_{22})}{b\pi_{22}}$.

Proof: See Theorem 4.2 in appendix.

In both of these equilibria player 1 separates. In (i) the separation is what could be termed natural in that one would expect that it would be $1(s_2)$ who would say d_1 given the structure of preferences and the values of the parameters. Given the low value of D, $2(s_1)$ disagrees with d_1 while type s_2 agrees with player 1. In the second equilibrium player 1 separates in the opposite manner. It is optimal for $1(s_1)$ to say d_1 secure in the knowledge that player 2 will correct an erroneous decision. However, additional restrictions on the cost of delay are necessary. In both of these equilibria the game ends after two stages and this is achieved because player 1 separates and so there is a larger information content in his message.

Proposition 4.9: There is one three stage equilibrium where both types of player 1 says d_0, $2(s_2)$ disagrees and so does $1(s_1)$ in the next stage. This equilibrium exists if $\pi_8 \geq \pi \geq \max\{\pi_9, \pi_6\}$ where

$$\pi_9 \equiv \frac{\dfrac{D^2a}{D^2a+b} - \pi_{12}}{\dfrac{D^2a}{D^2a+b} - \pi_{12} + \dfrac{(D-1)a(1-\pi_{11})}{D^2a+b}}$$

Proof: See Theorem 4.2 in appendix.

Here player 1's message does not reveal any information and so the burden falls on $2(s_2)$ to disagree and thus reveal his information. $1(s_1)$ realizes that saying d_0 raises the possibility of delay but still persists with it because if he said d_1 player $2(s_2)$ would agree. This term shows up in the numerator while the remaining term in the expression involves the cost of delay. Figure 4 shows the possible equilibria for $D \leq \tilde{D}$. The next proposition deals with the existence of pure strategy equilibria and we obtain as a corollary that a pure strategy equilibrium always exists for any value of D.

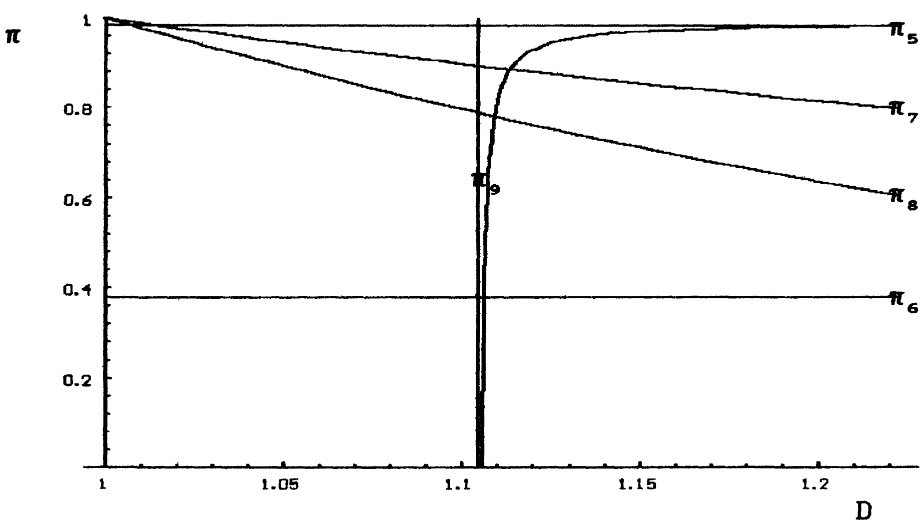

Figure 4. $\pi_{11} = 0.99$, $\pi_{12} = 0.5$, $\pi_{22} = 0.25$, $a = 45$, $b = 55$, $D \leq 1.25$.

Proposition 4.10: Either the equilibrium in Proposition 4.7 or equilibrium (i) in Proposition 4.8 always exists.

Proof: Note that $\pi_7 \geq \pi_8$ for $D \geq \frac{b\pi_{22}}{a(1 - \pi_{22})}$, a condition which is always satisfied since the maximum value π_{22} can attain is $\frac{a}{a + b}$. Consequently the maximum value that $\frac{b\pi_{22}}{a(1 - \pi_{22})}$ can reach is 1. However, $D > 1$, so the condition is always satisfied. So any π must satisfy the conditions for 4.7 or (ii) in 4.8.

Corollary 1: If $D \leq \tilde{D}$ a pure strategy Nash equilibrium always exists.

Corollary 2: A pure strategy equilibrium always exists for any value of D.

Proof: We get this result by combining Corollary 1 and the corollary to proposition 4.3.

In the equilibrium in proposition 4.7 the decision is reached without any delay and the information cost is the same as that in (13). In all other equilibria the correct decision is always reached but with varying degrees of costs of delay. In 4.8(i) the cost of delay is

$$(D-1)a(1-\pi_{12})\pi(1-\pi), \tag{17}$$

while that for 4.8(ii) and 4.9 are, respectively,

$$(D^2-1)a(1-\pi_{12})\pi(1-\pi) + (D-1)b\pi_{22}(1-\pi)^2, \tag{18}$$

and $(D-1)a(1-\pi_{11})\pi^2 + (D-1)a(1-\pi_{12})\pi(1-\pi) + (D-1)b\pi_{22}(1-\pi)^2$. (19)

A comparison of (17), (18) and (19) reveals that 4.8(i) is the best in terms of economizing on delay. Also, comparing (13) and (17) reveals that 4.7 is better than 4.8(i) if $\pi \geq \pi_7$. Then, as a counterpart to proposition 4.6 we get proposition 4.11.

Proposition 4.11: The efficient equilibria are those in proposition 4.7 and (i) in proposition 4.8.

Combining 4.6 and 4.11 we see that there are two characteristics of efficient equilibria. Either these involve decisions reached in one stage or player 1 separates so that his message is informative. Overall the difference in outcomes between $D \geq \tilde{D}$ and $D \leq \tilde{D}$ is that for $D \geq \tilde{D}$ it is not possible to reach the correct decision in any equilibrium though decisions will be reached quickly while if $D \leq \tilde{D}$ decision making will take longer but the correct decision is more likely to obtain. If the time between messages is reduced so that D tends to one the only equilibria which will remain are those in proposition 4.8. Thus the equilibrium will be efficient as the cost of delay goes down. Thus the Coase theorem holds as is stated in the next proposition.

Proposition 4.12: As D tends to one only efficient equilibria remain.

From looking at the Figures 3 and 4 it can be seen that there are a number of situations where there are multiple Nash equilibria. For example if $D \geq \tilde{D}$ if the condition $\pi_2 \geq \pi \geq \max\{\pi_3, \pi_4\}$ is satisfied there will be two pure strategy Nash equilibria, (ii) in proposition 4.2 and (i) in proposition 4.4. Results in the literature on there being an odd number of equilibria for almost all games suggest that in that case there will be a mixed strategy equilibria. Indeed, we can show that if $D \geq \tilde{D}$ and $\min\{\pi_1, \pi_2\} \geq \pi \geq \max\{\pi_3, \pi_4\}$ then (see Theorem 4.3 in appendix) there is an equilibrium where $1(s_2)$ mixes between saying d_0 and d_1 while $2(s_2)$ mixes between agreeing and disagreeing on d_0. Since $D \geq \tilde{D}$ both players would immediately agree in the next stage following a disagreement. Player $1(s_1)$ says d_0 and $2(s_1)$ agrees with player 1. The probability with which $1(s_1)$ says d_0 is given by

$$\frac{\pi}{1 - \pi} \frac{\pi_{12} - \dfrac{a}{a + Db}}{\dfrac{a}{a + Db} - \pi_{22}}. \tag{20}$$

and the probability with which player $2(s_2)$ agrees with d_0 is given by

$$\frac{\pi}{1 - \pi} \frac{\pi_{12} - \dfrac{a}{a + b}}{\dfrac{a}{a + Db} - \pi_{22}} - \frac{(D - 1)b\pi_{22}}{\dfrac{a}{a + Db} - \pi_{22}} \tag{21}$$

This equilibrium can be regarded as a mixture of outcomes between the equilibria in proposition 4.2 and equilibrium (i) in proposition 4.4. From our earlier discussion we know that the equilibria in proposition 4.2 are the most efficient given the high cost of delay. This equilibrium can be seen as an attempt to alleviate inefficiency. From (19) the probability of saying d_0 is the ratio of the losses from saying d_0 and d_1 weighed by the probabilities of player 1 receiving the signals s_1 and s_2. An examination of (19) reveals that the probability of saying d_0 increases with D and π. Similarly from (20) we can see that the probability for agreeing goes up with an increase in π and D. Thus this mixed strategy equilibrium converges to equilibrium (i) in proposition 4.2 which we know is efficient.

Similarly, if $D \leq \tilde{D}$ and $\min\{\pi_7, \pi_8\} \geq \pi \geq \max\{\pi_6, \pi_9\}$ then there is a mixed strategy equilibrium (Theorem 4.4 in appendix) where $1(s_1)$ and $2(s_1)$ say d_0 in every stage. Player $1(s_2)$ mixes between saying d_0 and d_1 and player $2(s_2)$ mixes between agreeing and disagreeing. The probability with which player $1(s_2)$ says d_0 is given by

$$\frac{\pi}{1-\pi}\,\frac{(D^2-1)a(1-\pi_{12})}{a(1-\pi_{22}) - Db\pi_{22}} \tag{22}$$

and the probability of agreeing is given by

$$\frac{(D-1)\{\pi a(1-\pi_{12}) - (1-\pi)b\pi_{22}\}}{(1-\pi)\{a(1-\pi_{22}) - Db\pi_{22}\}}. \tag{23}$$

As in the discussion of our earlier mixed strategy equilibrium we can see that this equilibrium is a mixture of the outcomes in the equilibrium in proposition 4.7, (i) in proposition 4.8 and that in proposition 4.9. Since the equilibrium in proposition 4.9 is less efficient than the other equilibria this equilibrium can be seen to improve matters. Even though we have not made an exhaustive investigation of all possible mixed strategy equilibria we would argue that mixed strategy equilibria in this setting are plausible and can actually be beneficial.

The message which comes out of this section is that it is possible that better decisions will be reached if the individuals within a team are patient. Contrary to expectations it is not the case that better decisions are reached if the cost of delay are lowered. If costs are to be lowered they have to be lowered below $\tilde{D}$. Otherwise there are equilibria which achieve the same outcomes as one-stage equilibria but with more communication and delay. These equilibria would in some measure justify the belief that in some cases institutions such as committees do not reach better decisions and only serve to spend time in fruitless discussions. In the next section we will we will look at equilibria when individuals do not have identical preferences with regard to losses from different outcomes.

5. Biases in preferences

Casual introspection would suggest that as a general rule individuals in a team rarely agree on the decisions to be reached which would suggest that the contents of the previous section, while being interesting in its own right, is probably not very relevant. We would argue that such doubts are misplaced because it is important to know how strongly the individuals disagree. Let us assume that 1's losses from the decisions d_0 and d_1 are a_1 and b, while those of 2 for the same decisions are a_2 and b. This difference is enough to generate different preferences over outcomes and though we could have assumed different preferences on the loss associated with reaching a decision d_1 this is not necessary.

We could investigate the equilibria under the condition that

$$\pi_{11} > \pi_{12} > \frac{a_2}{a_2 + b} > \frac{a_1}{a_1 + b} > \pi_{22} \qquad (24)$$

but the results would not be very different from those in the previous section. If the above condition holds then both players agree on the decisions to be reached for the different events. Player 1 prefers d_0 more strongly than player 2 and that would change the values of the parameters for which the equilibria in the previous section hold but the set of equilibria would not change its important characteristics. Another condition we could investigate is

$$\frac{a_2}{a_2 + b} > \pi_{11} > \pi_{12} > \pi_{22} > \frac{a_1}{a_1 + b} \qquad (25)$$

Here, players 1 and 2 disagree on the decisions to be reached in all events and there is no scope for communication nor would it help since no new information would change peoples' minds as to what decision ought to be reached. This is similar to a bargaining problem and we would argue that there is an element of bargaining in team decision making but that is not its sole feature. The above specification would leave no room for investigation of the communication aspect.

Instead we will also assume that

$$\pi_{11} > \frac{a_2}{a_2 + b} > \pi_{12} > \frac{a_1}{a_1 + b} > \pi_{22}. \qquad (26)$$

Thus 1 and 2 have the same preferences over the events (s_1,s_1) and (s_2,s_2). They differ on the event (s_1,s_2); 1 would like the decision to be d_0 while 2 prefers d_1. The situation is shown in Figure 5. Thus there are possibilities for both bargaining and communication. These preferences are common knowledge so that 1 and 2 are aware of their opponent's biases. The out of equilibrium beliefs are the same as in section 3[2].

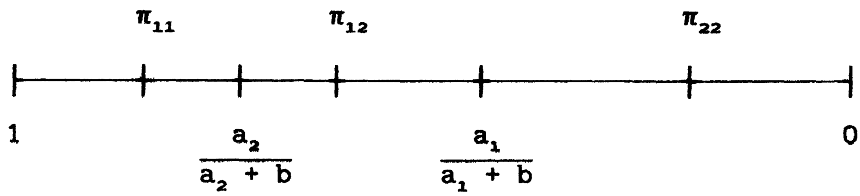

Figure 5

In section 4 we conducted our investigation of the possible equilibria by dividing up the possible values of the cost of delay, D, into two: (i) $D \geq \tilde{D}$ and (ii) $D < \tilde{D}$. Recall that for $D \geq \tilde{D}$ the players found that it was not worthwhile to incur delay to get the decision d_0 for the event (s_1,s_2). In this section the players are biased and disagree over the decision in case of this particular event. We will denote

$$\bar{D} \equiv \frac{b\pi_{12}}{a_1(1-\pi_{12})} \tag{27}$$

and $$\hat{D} \equiv \frac{a_2(1 - \pi_{12})}{b\pi_{12}}. \tag{28}$$

For $D \geq \bar{D}$ player 1, who prefers d_0 for (s_1,s_2) will not find it worthwhile to insist on d_0 even if player 2 were to concede in the next stage. Similarly, for $D \geq \hat{D}$ player 2 will concede to d_0 even though he prefers d_1.

To look at the possible equilibria we consider four possible ranges of D, (i) $D \geq \max\{\bar{D}, \hat{D}\}$, (ii) $\bar{D} \geq D \geq \hat{D}$, (iii) $\hat{D} \geq D \geq \bar{D}$ and (iv) $D \leq \min\{\bar{D}, \hat{D}\}$. In (i) both 1 and 2 find that in the event (s_1,s_2) it is not worth the while to send the game into the next stage to get their desired outcome. In (ii) and (iii) one individual finds D small enough to insist on their desired outcome while in (iv) both individuals find that D is small enough to worth insisting on their desired outcome provided the other individual concedes. The set of equilibria for these four cases are shown in Theorem 5.1, 5.2, 5.3 and 5.5 in the appendix. The proofs are similar to the proofs for

Theorem 4.1 and 4.2 and are not shown. Before we go on to discuss the equilibria we will prove that for the first three cases the game will end within three stages.

Proposition 5.1: Let K be the last stage where there is a disagreement, then, for $D \geq \bar{D}$ or $D \geq \hat{D}$ or both, $K \leq 3$.

Proof: Suppose K = 4; there are two possible sequences in which this could occur (i) $\{d_0,d_1,d_0,d_1,d_0,d_0\}$ and (ii) $\{d_1,d_0,d_1,d_0,d_1,d_1\}$. Consider (i) when 1 plays d_0 in stage one $2(s_1)$ would agree immediately if $D \geq \hat{D}$. So if 2 plays d_1 again 2 must be type s_2. If 1 says d_0 again then 1 must be type s_1 and $D \leq \bar{D}$. Then 2 should agree with 1 since 1 will always say d_0. If $D \leq \hat{D}$, then $2(s_2)$ would always say d_1 so the fact that 2 has agreed with d_0 in stage 4 indicates that 2 is type s_1. If 1 persists with d_0 1 must be type s_1 given that $D \geq \bar{D}$. Then $2(s_1)$ should agree with d_0 in the third stage. For (ii) if 2 says d_0 when 1 says d_1 then 2 must be s_1 so that both types of 1 should immediately agree with 2.

We will now go on to investigate the possible equilibria for each of the values of D which we have described. We begin with a proposition on the equilibria for case (i). It is straight forward to show that, as a counterpart to Proposition 4.1, the game must end within two stages.

Proposition 5.2: The set of equilibria for $D \geq \max\{\bar{D},\hat{D}\}$ is the same as that in Propositions 4.2 and 4.4 with the values of the parameters π_1, π_3, π_4, π_5, π_6 replaced by β_{11}, β_{12}, β_{13}, β_{14}, β_{15} respectively and π_2 with β_{21}.

The value of these expressions are shown along with the corresponding theorems in the appendix. There is a close correspondence between the

π-terms in section 4 and the β-terms here. All of these expressions can be derived by replacing the term a in section 4 with a_1 or a_2 depending on whether the condition is relevant for player 1 or 2. Even though the set of equilibrium strategies are the same there are important differences.

Consider the first equilibrium in proposition 4.2 where both types of player 1 say d_0 and player 2 agrees. This same equilibrium exists here if $\pi \geq \max\{\beta_{11},\beta_{21}\}$. The condition $\pi \geq \beta_{11}$ is necessary for $1(s_2)$ to say d_0 and $\pi \geq \beta_{21}$ for $2(s_2)$ to agree with player 1. In section 4 the corresponding condition is $\pi \geq \max\{\pi_1,\pi_2\}$ and we showed that $\pi_1 \geq \pi_2$ so that the only condition necessary to achieve this equilibrium is $\pi \geq \pi_1$. Figure 6 shows that this is no longer the case. The values of the parameters in figure 6 are the same as that in figure 3: the only addition is $a_2 = 70$. Now it is possible for β_{21} to be greater than β_{11}; for low values of D, $2(s_2)$ would not be willing to agree with 1 when he says d_0. Thus disagreements are more likely with biased individuals which should come as no great surprise. A glance at β_{21} will reveal that it increases with a_2 and so this equilibrium is less likely to obtain the more biased is player 2 in favor of d_1. The condition for the other one-stage equilibrium, where player 1 separates with $1(s_1)$ saying d_0 and $1(s_2)$ saying d_1, remains the same as before.

Also, in section 4 two-stage equilibria could only exist along with one stage equilibria. From figure 6 we can see that this property does not extend to the case of different preferences. For $\pi \geq \max\{\beta_{11},\beta_{12},\beta_{13}\}$ and $\pi \leq \beta_{21}$ the only equilibria is one where both types of player 1 say d_0 and player $2(s_2)$ disagrees. In the next stage both types of player 1 agree with player 2 if confronted by a disagreement. In this example for values of D less than 3 and π between π_{14} and π_{15} we will get the last equilibrium of the earlier section. In this equilibrium player $1(s_1)$ says d_1 and $1(s_2)$ says d_0; $2(s_1)$ disagrees on d_1 as does $2(s_2)$ for d_0. Again, for high values of π this equilibrium will exist without being accompanied by a one-stage equilibrium.

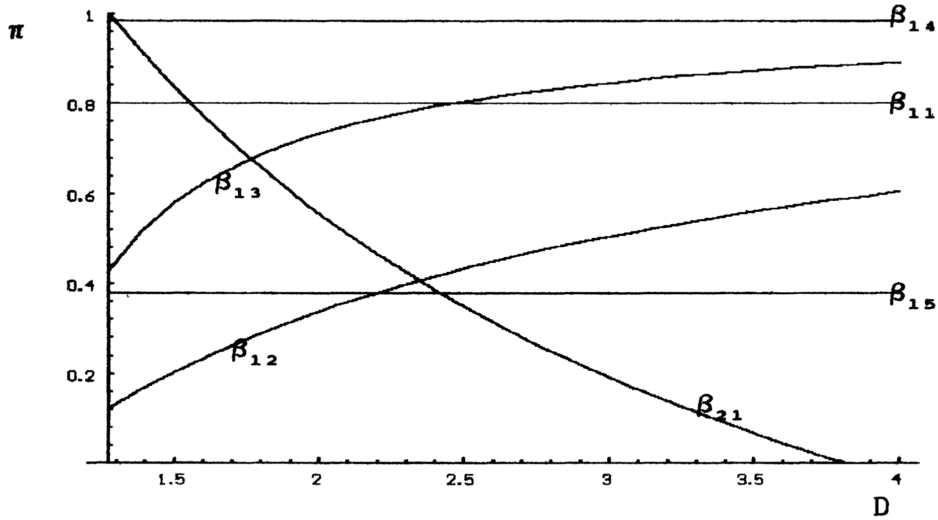

Figure 6. $\pi_{11} = 0.99$, $\pi_{12} = 0.5$, $\pi_{22} = 0.25$, $a_1 = 45$, $a_2 = 70$, $b = 55$, $D \geq 1.27$.

Figure 6 shows a situation where a pure strategy Nash equilibrium always exists, but as Figure 7 shows a pure strategy Nash equilibrium may not exist. In Figure 7 if $\beta_{11} \leq \pi \leq \min\{\beta_{15},\beta_{12},\beta_{21}\}$ a pure strategy Nash equilibrium will not exist. The above condition means that if $2(s_2)$ were to respond by d_1 to 1's d_0 then both types of 1 would rather say d_1. If 2 were to agree with 1 when 1 said d_0 both types of 1 would say d_0. If both types of 1 did say d_0, i.e., they were to pool on d_0, $2(s_2)$ would respond with d_1. However, if player $2(s_2)$ does say d_1 when player 1 says d_0 player 1 would rather say d_1; but if player 1 were to pool on d_1 and d_0 were observed player 2 would perceive that 1 is type s_1 and hence agree if 1 were to say d_0. Thus given the out of equilibrium beliefs a pure strategy Nash equilibrium does not exist. Of course there will be a mixed strategy equilibrium and by virtue of Proposition 5.1 the game would end within three stages. However, a mixed strategy equilibrium must involve some disagreement by the very nature of the game and this would reinforce our notion that biases give rise to more disagreement.

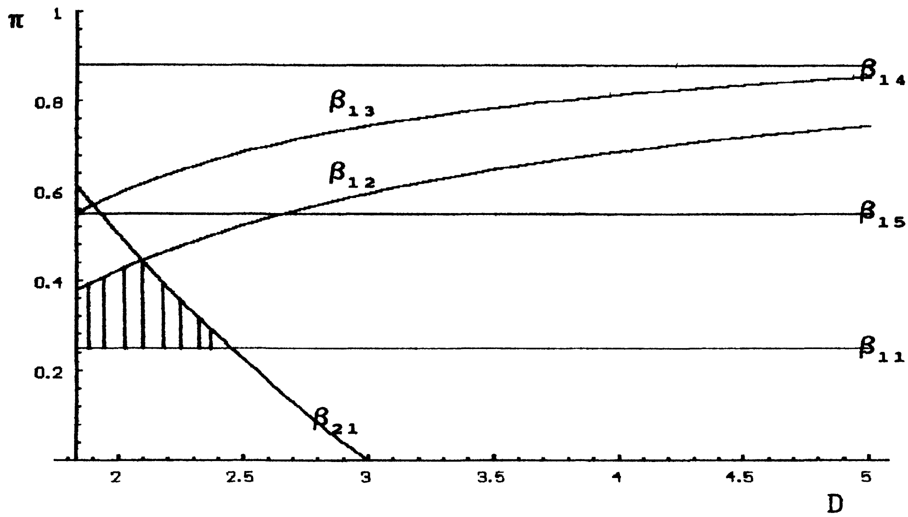

Figure 7. $\pi_{11} = 0.9$, $\pi_{12} = 0.6$, $\pi_{22} = 0.4$, $a_1 = 9/11$, $a_2 = 2$, $b = 1$, $D \geq 1.83$

The possibility for increased disagreement could lead us to expect that there would be more inefficiency when individuals are biased. By analogy with Proposition 4.7 we could expect that one-stage equilibria are the most efficient equilibria but since the two players are biased it is possible that in comparing a one-stage equilibrium with a two stage equilibrium the two individuals might reach different conclusions regarding which one is better. The total costs for each of the four possible equilibria are the same as that in (13)-(16) with a being replaced by a_1 and a_2 depending on which player we want to consider. A comparison of the costs of the various equilibria show that it is still true that one-stage equilibria are more efficient than two-stage equilibria. In our previous discussion we have shown that it is possible that for some values of the parameters only two-stage equilibria will exist and so we conclude that if the players are biased then the equilibria are more likely to be inefficient.

We have mentioned earlier that players 1 and 2 disagree over the best decision to be reached in the event (s_1,s_2). One approach while investigating the various equilibria would be to ask to what extent is player 1 able to reach his desired outcome. We would surmise that this task would be facilitated if player 1 is more biased or is more patient. Proposition 5.3 investigates this possibility and shows that the equilibrium strategies for player 2 remain the same as in Proposition 5.2.

Proposition 5.3: The set of equilibria for $\bar{D} \geq D \geq \hat{D}$ can be derived by replacing the strategies of Player $1(s_1)$ saying d_0 once and agreeing in the next stage in proposition 5.2 with $1(s_1)$ saying d_0 in every stage.

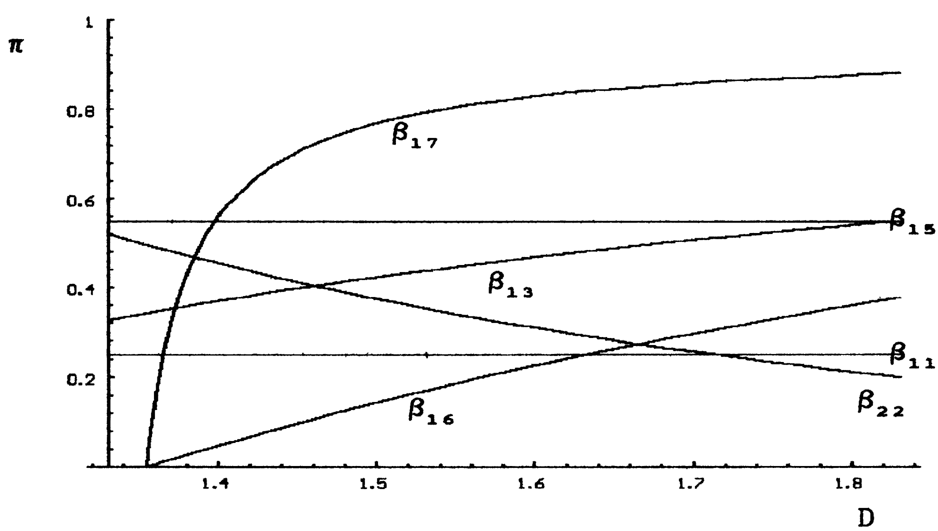

Figure 8. $\pi_{11} = 0.9$, $\pi_{12} = 0.6$, $\pi_{22} = 0.4$, $a_1 = 9/11$, $a_2 = 2$, $b = 1$, $D \geq 1.33$.

The equilibria are shown in Figure 8 and the values of parameters are the same as in figure 7. In the previous example player $2(s_2)$ would find it worthwhile to disagree if $\pi \leq \beta_{21}$ while here the similar condition is $\pi \leq \beta_{22}$. Consequently there is more disagreement but at the same time player 1 is more likely to get his preferred outcome. The shaded portion show the values of π and D for which there are no pure strategy equilibria.

The situation is drastically changed if player 2 is made more patient or becomes more biased. The results are shown in Proposition 5.4 and Figure 9. The value of β_{14} which is high is not shown so that the other terms can be shown properly. First, both types of player 1 saying d_0 and player 2 agreeing is no longer an equilibrium. In fact, in all the equilibria player 2 disagrees with 1 if 1 says d_0. Also, for the first time we get new equilibria which involve both types of player 1 saying d_1; a result of the lower value of D which now allows player 2 to insist on d_1.

Proposition 5.4: (i) There is a one-stage equilibrium where player 1 says d_1 and player 2 agrees. (ii) There are three two-stage equilibria: (a) player 1 says d_0, $2(s_2)$ disagrees; (b) player 1 says d_1, player $2(s_1)$ disagrees; (c) player $1(s_1)$ says d_0, $1(s_2)$ says d_1 and $2(s_2)$ disagrees.

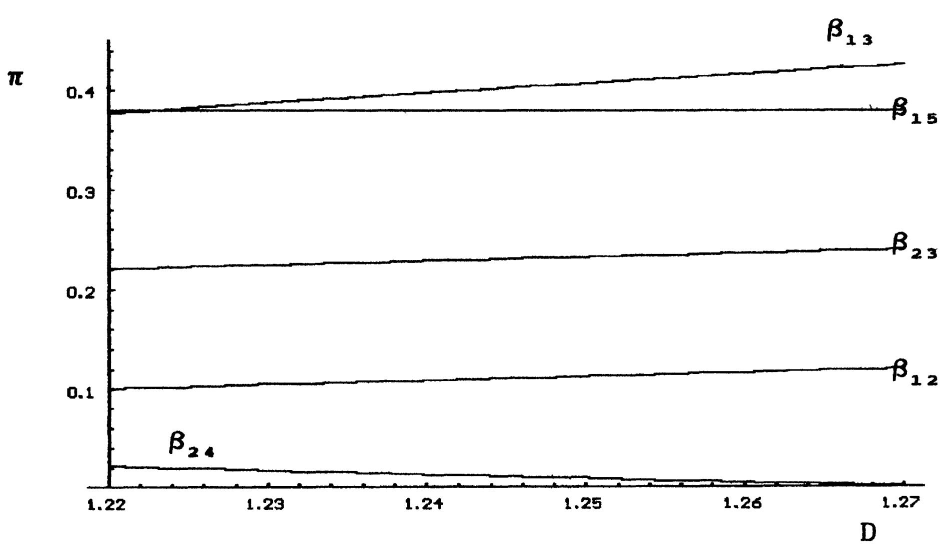

Figure 9. $\pi_{11} = 0.99$, $\pi_{12} = 0.5$, $\pi_{22} = 0.25$, $a_1 = 45$, $a_2 = 70$ $b = 55$, $D \geq 1.22$.

The shift in fortunes of player 2 between Theorems 5.1, 5.2 and 5.3 suggest that the player who moves first has an advantage. This advantage is strengthened if the player has a stronger bias. For example if $D \geq \max\{\bar{D},\hat{D}\}$, and player 2 starts the game, we can show that equilibria where player 1 pools on d_0 no longer exist. Equilibria involving player 1 separating in two different ways exist but for different values of the parameters and we have two other equilibrium where both types of player 2 start the game by sending the message d_1. The equilibria are shown in Theorem 5.4 without the constraints on the parameters.

There is a fair degree of symmetry between Theorem 5.1 and 5.4. In both we get equilibria which involve the player who starts the game pooling on the decision towards which they are biased and the other player agreeing. There is an equilibrium where player 1 separates with $2(s_1)$ saying d_0, $2(s_2)$ saying d_1 and player 2 agrees. If we strengthen 2 by making $D \leq \hat{D}$ while retaining $D \geq \tilde{D}$ we will have the same equilibria but 2 will be able to get the decision of his choice for higher values of π and D. Reversing the constraints on D will produce equilibria similar to Theorem 5.3.

The last condition on D for which we will look for equilibria is $D \leq \min\{\bar{D},\hat{D}\}$. The results are shown in Proposition 5.5. The number of possible equilibria increases and the comprise all the equilibria from Theorem 5.2 and 5.3 with the exception of (IV) from Theorem 5.2. The excluded equilibrium involves $1(s_1)$ saying d_1 and $1(s_2)$ saying d_0 and $2(s_1)$ agreeing with d_0. Since $2(s_1)$ prefers d_1, and D is low, agreeing with d_0 is no longer optimal.

Proposition 5.5: The equilibria for $D \leq \min\{\bar{D},\hat{D}\}$ are the same as those in

Proposition 5.2 and 5.3.

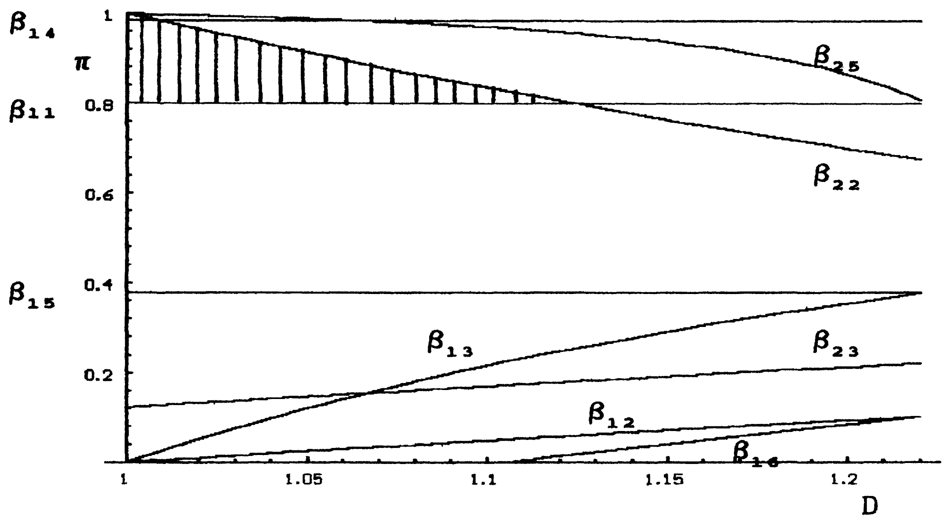

Figure 10. π_{11} = 0.99, π_{12} = 0.5, π_{22} = 0.25, a_1 = 45, a_2 = 70, b = 55, D ≤ 1.22.

From our earlier discussion there would still be a first mover advantage but with the multiplicity of equilibria this will be diminished. Since D is small the two players are more or less equals in strength of bargaining power. This raises the specter of inefficient haggling over decisions and the next focus of inquiry will be on the possibility of this occurring. As can be seen from Figure 10 and proposition 5.6 there are values of D and π for which there are no pure strategy equilibria. There will of course be mixed strategy equilibria but we can no longer be sure that the game will end within three stages. Proposition 5.7 shows the structure of mixed strategy equilibria.

Proposition 5.6: If $\beta_{22} \geq \pi \geq \max\{\beta_{15},\beta_{11}\}$ there is no pure strategy equilibrium.

Proof: The proof follows from noting that this condition does not satisfy any of the conditions for the existence of an equilibrium in Theorem 5.5.

If this condition is satisfied then both types of player 1 would say d_0 if player 2 would agree with d_0. However, if both types of player 1 did say d_0, both types of player 2 would disagree. The condition $\pi \geq \beta_{15}$ ensures that player 1 does not separate with $1(s_1)$ saying d_1 and $1(s_2)$ saying d_0, which is a possible pure strategy equilibrium.[3]

Proposition 5.7: If

$\sigma_1(d_0|h^k,\mu(s_1|h^k),s_1) = 1, \forall k \leq x; \sigma_1(d_0|h^k,\mu(s_1|h^k),s_1) = 0, \forall k > x$

$\sigma_1(d_0|h^k,\mu(s_1|h^k),s_2)$

$\sigma_2(d_0|h^k,\mu(s_1|h^k),s_1)$

$\sigma_2(d_0|h^k,\mu(s_1|h^k),s_2) = 0, \forall k \leq y; \sigma_2(d_0|h^k,\mu(s_1|h^k),s_2) = 1, \forall k > y$

is a equilibrium, then

(a) $\sigma_1(d_0|h^k,\mu(s_1|h^k),s_2) = 1$ and $k > 0 \Rightarrow \sigma_1(d_0|h^\tau,\mu(s_1|h^\tau),s_2) = 1 \ \forall \tau > k$.

(b) $\sigma_2(d_0|h^k,\mu(s_1|h^k),s_1) = 0 \Rightarrow \sigma_2(d_0|h^\eta,\mu(s_1|h^\eta),s_1) = 0 \ \forall \eta > k$.

(c) $\exists \tau, \eta$ s.t. $\sigma_1(d_0|h^{\tau+k},\mu(s_1|h^{\tau+k}),s_2) = 0, \sigma_2(d_0|h^{\eta+k},\mu(s_1|h^{\eta+k}),s_1) = 1$ $\forall k$.

(d) Let $K_{12} = \max\{k: \sigma_1(d_0|h^k,\mu(s_1|h^k),s_2) > 0\}$ and

$K_{21} = \max\{k: \sigma_2(d_1|h^k,\mu(s_1|h^k),s_1) > 0\}$.

Then $K_{21} + 1 \geq K_{12} \geq K_{21} - 1$.

(e) Either $x = K_{12}$ or $K_{21} + 1$ and $y = \infty$, or $x = \infty$ and $y = K_{21}$ or $K_{12} + 1$.

Part (a) says that if $1(s_2)$ says d_0 with probability one in any stage beyond the first stage then he always says d_0 from then on. Suppose $1(s_2)$ says d_0 with probability one in some stage k, then $2(s_1)$ has a choice between saying d_1 or ending the game by saying d_0. Ending the game now dominates ending the game after one stage since no delay will take place and no added information about 1 is available as both types play d_0 with probability one. So $2(s_1)$ should have said d_0 with probability one in stage $k - 1$. If he has not then $2(s_1)$ will play d_1 with probability one in stage $k + 1$. Consequently $1(s_2)$ will play d_0 with probability one in stage $k + 2$. He has a choice of playing d_1 and ending the game or playing d_0 and prolonging the game. If he plays d_1 then he should have done so in stage k so he must continue to say d_0. By a similar argument $2(s_2)$ should continue to say d_1. This argument does not hold for stage zero which is therefore an exception. Part (b) makes a similar statement about $2(s_1)$. Part(c) states that $1(s_2)$ and $2(s_1)$ would not continue in the game for ever.

Part (d) states that $2(s_1)$ will play d_0 and end the game earlier than $1(s_2)$. As the game continues $2(s_1)$ will eventually become so pessimistic about the possibility of 1 being of type 2 that he will concede the game. Then $1(s_2)$ will concede the game in the stage after that. Conceding earlier cannot constitute an equilibrium because $2(s_1)$ should then say d_1 with probability one in the present stage and by parts (a) and (b) that cannot be an equilibrium. Thus the first mover advantage is still present. Once the types are revealed one of the two players have to concede the game. If it is $1(s_1)$ who concedes then once K_{21} has passed and it is clear that player 2 is type 2, $1(s_1)$ should also concede. This is shown in part (e). If $2(s_2)$ is to concede then he should do so after the stage in which $1(s_2)$ concedes. Proposition 5.6 is very similar to Proposition 2 in Chatterjee and Samuelson(1987). Part (e) restates the argument in the proof of Theorem 5.4. The structure of the equilibrium is similar to that in bargaining games. Usually, in multistage bargaining there are a series of decreasing price offers until one is accepted. Here $1(s_2)$ and $2(s_1)$ insist on their preferred outcomes but with decreasing insistence until 2's type is revealed and the game then ends within the next two stages.

Our final concern has to deal with the Coase conjecture which proposes that if the time spent between each message is low then the members of a team should be able to reach an efficient decision. This would be valid for section 4 where we could show that as D goes to zero so does the possible inefficiency. That is not true when the individuals are biased. Lowering D raises the possibility of an equilibrium which takes more than three stages and lowering D any further would increase the number of stages the two individuals disagree. This has to inefficient because whatever the preferences of the social planner these could be implemented with a pure strategy equilibrium which takes only three stages. A comparison of costs in case of Proposition 5.2 and 5.3 show that the equilibria there are efficient. We earlier showed that there are inefficient equilibria if $D \geq \max\{\bar{D},\hat{D}\}$ and so lowering D does lead to better equilibria but lowering D much further may not do so.

6. Conclusion

This paper investigates the possible equilibria in a two-person game of information transmission. The issue at stake is how efficiently is information aggregated across individuals and this issue is addressed from the viewpoint of efficiency of information aggregation and efficiency in terms of time taken to reach a decision. We have considered a simple model where there are only two individuals and who receive only two signals regarding the desirability of their two decisions. Obviously, the model could be extended in different directions, but, within this simple framework, it is possible to show the impact of disagreement among the team members over outcomes and how it can introduce delay and also show how the cost of delay impacts on team decisions. These results should prove significant in studying the structure of organizations in particular and such is the intended direction of further research.

Footnotes

1. The condition required for $1(s_1)$ to say d_1 is
$b\pi_{11}\pi + b\pi_{12}(1 - \pi) \le a(1 - \pi_{11}) + Db\pi_{12}(1 - \pi)$.
The left hand side shows the payoff from $\sigma_1(d_0|h^o,s_1) = 0$, $\sigma_1(d_0|\{d_1,d_0\}, s_1) = 1$ and the right hand shows the payoff from $\sigma_1(d_0|h^o, s_1) = 1$, $\sigma_1(d_0|\{d_0,d_1\}, s_1) = 0$. Similarly, the condition for $1(s_2)$ to follow the same strategy is
$b\pi_{12}\pi + b\pi_{22}(1 - \pi) \le a(1 - \pi_{12}) + Db\pi_{22}(1 - \pi)$.
For $2(s_1)$ $\sigma_2(d_0|\{d_0\},s_1) = 1$ is a dominant strategy while $\sigma_2(d_0|\{d_1\},s_1) = 0$ is optimal if
$b\pi_{11}\pi + b\pi_{12}(1 - \pi) \le D(a(1 - \pi_{11}) + a(1 - \pi_{12})(1 - \pi))$.
$2(s_2)$ finds $\sigma_2(d_0|\{d_1\},s_2) = 0$ optimal if
$b\pi_{12}\pi + b\pi_{22}(1 - \pi) \le D(a(1 - \pi_{12}) + a(1 - \pi_{22})(1 - \pi))$.
To sustain the equilibrium strategy of $\sigma_2(d_0|\{d_0\},s_2) = 0$, given the out of equilibrium belief $\mu(s_1|\{d_0\}) = 0$, we require $D \le \frac{a(1 - \pi_{22})}{b\pi_{22}}$. If, on the other hand, player 2 believes $\mu(s_1|\{d_0\}) = 1$, then $1(s_1)$ would play the strategy $\sigma_1(d_0|h^o, s_1) = 1$, because that would be the dominant strategy given that both types of 2 would agree with d_0 by player 1. Then this equilibrium does not agree with the intuitive criterion and should be discarded.

2. The equilibrium which produces this outcome is
$\sigma_1^k(d_0|h^k,s_t) = 1$ for $h^k = h^o$ and $h^k = \{h^{k-1},d_0\}$, $t = 1,2$.
$\sigma_1^k(d_0|h^k,s_t) = 0$ for $h^k = \{h^{k-1},d_1\}$, $t = 1,2$.
$\sigma_2^k(d_0|h^k,s_t) = 1$ for $h^k = \{h^{k-1},d_0\}$, $t = 1,2$.
$\sigma_2^k(d_0|h^k,s_t) = 0$ for $h^k = \{h^{k-1},d_1\}$, $t = 1,2$.
with out of equilibrium beliefs
$\mu(s_1|h^k) = 1$ for $h^k = \{h^{k-1},d_0\}$
$\mu(s_1|h^k) = 0$ for $h^k = \{h^{k-1},d_1\}$.
The question we, again, ask is what should the values of the parameters be for this equilibrium to hold. Note that $\sigma_1^o(d_0|h^o,s_1) = 1$ is a dominant strategy. The payoff from $\sigma_1^o(d_0|h^o,s_2) = 1$ is
$a_1(1 - \pi_{12})\pi + a_1(1 - \pi_{22})(1 - \pi)$,
while that from $\sigma_1^o(d_0|h^o,s_2) = 0$ is
$b\pi_{12}\pi + b\pi_{22}(1 - \pi)$.
So that $\sigma_1^o(d_0|h^o,s_2) = 1$ is optimal if
$a_1(1 - \pi_{12})\pi + a_1(1 - \pi_{22})(1 - \pi) \le b\pi_{12}\pi + b\pi_{22}(1 - \pi)$, or,

$$\pi \geq \beta_{11} \equiv \frac{\frac{a_1}{a_1 + b} - \pi_{22}}{\pi_{12} - \pi_{22}}$$

If 1 says d_0 and 2 says d_1 then we require 1 to believe that 2 is type s_2, $\mu(s_1|\{d_0,d_1\}) = 0$. Then the payoff from $\sigma_1(d_0|\{d_0,d_1\},s_1) = 0$ is $Db\pi_{12}$ while that from $\sigma_1(d_0|\{d_0,d_1\},s_1) = 1$ is $D^2a_1(1 - \pi_{12})$. So we would require

$Db\pi_{12} \leq D^2a_1(1 - \pi_{12})$, or $D \geq \bar{D} \equiv \frac{b\pi_{12}}{a_1(1-\pi_{12})}$.

Given the out of equilibrium beliefs $\sigma_1(d_0|\{d_0,d_1\},s_2) = 0$ is a dominant strategy.

The payoffs from $\sigma_2(d_0|\{d_0\},s_1) = 1$ and $\sigma_2(d_0|\{d_0\},s_1) = 0$ are
$a_2(1 - \pi_{11})\pi + a_2(1 - \pi_{12})(1 - \pi)$ and
$D\{b\pi_{11}\pi + b\pi_{12}(1 - \pi)\}$ respectively. Thus we need
$a_2(1 - \pi_{11})\pi + a_2(1 - \pi_{12})(1 - \pi) \leq D\{b\pi_{11}\pi + b\pi_{12}(1 - \pi)\}$, or

$$\pi \geq \beta_{24} \equiv \frac{\frac{a_2}{a_2 + Db} - \pi_{12}}{\pi_{11} - \pi_{12}}$$

Similarly, we can show that $\sigma_2(d_0|\{d_0\},s_1) = 1$ is optimal if

$$\pi \geq \beta_{21} \equiv \frac{\frac{a_2}{a_2 + Db} - \pi_{22}}{\pi_{12} - \pi_{22}}.$$

From (26) $\sigma_2(d_0|\{d_1\},s_t) = 0$ is a dominant strategy for t = 1, 2. The final condition we require is derived from consideration of player 2's payoffs from his strategies in the history $\{d_0,d_1,d_0\}$. Our stipulation of beliefs requires player 2 to believe that he is facing s_1 and his payoff from agreeing at this stage is $D^2a_2(1 - \pi_{12})$ and that from d_1 is $D^3b\pi_{12}$ and saying d_0 would be optimal if

$D \geq \hat{D} \equiv \frac{a_2(1 - \pi_{12})}{b\pi_{12}}$.

Therefore, if the above conditions hold we will have our equilibrium. Actually, if the last condition holds then $\pi \geq \beta_{24}$ is automatically satisfied. The reader can check that our arguments about the out of equilibrium beliefs still hold and are, indeed, strengthened. A significant difference for before is that $2(s_1)$ agreeing with d_0 is no longer a dominant strategy and depends on whether $D \geq \hat{D}$.

3. Note that for other values of π there could be multiple Nash equilibria and, consequently, mixed strategy equilibria. However, the mixed strategy equilibria will involve a mixture of the pure strategy equilibria and the game will end within three stages.

APPENDIX

For the purpose of writing down the possible equilibrium it would be easier to write down strategies in the pure reduced normal form. This would lessen the burden of notation since we would not have to write down strategies for histories which would not be reached. We would caution though the concept of sequential equilibrium is valid only in the extensive form and when we write down a particular equilibrium we have at the back of our mind a particular sequential equilibrium. Player i type t's pure strategy is a double $\{n^d_{it}\}$, d = 0,1 for $t \in \{1,2\}$, $i \in \{1,2\}$.

Player i's strategy is the number of periods he is going to say $d_0(d_1)$ before ending the game by saying $d_1(d_0)$. A period is defined to be two stages. Thus stage 0 and 1 comprise the first period. $n^d_{it} = n$, implies that player i is going to say d_0 for n-1 periods and d_1 in the nth period. There is an asymmetry between player 1 and 2 in that player 2 gets to observe player 1's choice of message in the first period before he makes a move. We will denote a mixed strategy as δ^{dn}_{it}. This, is the probability with which player i of type t plays the pure strategy $n^d_{it} = n$. The payoff from the pure strategy n^o_{it} is

$$S(n^o_{1t}) \equiv S^{on}_{1t} = \sum_{r=1}^{2} p(s_r) \sum_{\nu=1}^{n-1} \delta^{1\nu}_{2t}\, a_1[1 - \pi_{tr}]D^{2(\nu-1)} + [1 - \sum_{\nu=1}^{n-1} \delta^{1\nu}_{2t}]\, b_1\pi_{tr}D^{2n-1}$$

The payoff from n^1_{1t} is similar with $a_1[1 - \pi_{tr}]$ being replaced by $b_1\pi_{tr}$ and vice versa. Also, the payoff for player 2 should reflect the fact that he follows player 1 in that the power on D accompanying the first term should become $2\nu - 1$ and that in the second term 2(n - 1). A mixed strategy is denoted δ^{dn}_{it} which is the probability with which player i plays the pure strategy $n^d_{it} = n$. Thus,

$$u(\{\delta_{it}^{dn}\}) = \sum_{d=0}^{1} \sum_{n=1}^{\infty} \delta_{it}^{dn} S_{it}^{dn}$$

A Nash equilibrium is a pair $\{\delta_{it}^*, \delta_{jr}^*\}$ such that

$$u(\delta_{it}^*, \delta_{jr}^*) \geq u(\delta_{it}, \delta_{jr}^*)$$

$$u(\delta_{it}^*, \delta_{jr}^*) \geq u(\delta_{it}^*, \delta_{jr})$$

A pure strategy equilibrium is going to be a eight-tuple $\{n_{it}^d\}$; $d = 0,1$; $t = 1,2$; $i = 1,2$.

Theorem 4.1: For $D \geq \tilde{D}$, there are four possible types of pure strategy equilibria. These are

(I) {2, 1, 2, 1, 1, 1, 1, 1} for $\pi \geq \max\{\pi_1, \pi_2\}$,

(II) {2, 1, 1, 2, 1, 1, 1, 1} for $\pi \leq \pi_1$,

(III) {2, 1, 2, 1, 1, 1, 1, 2} for $\pi_2 \geq \pi \geq \max\{\pi_3, \pi_4\}$,

(IV) {1, 2, 2, 1, 2, 1, 1, 2} for $\pi_5 \geq \pi \geq \pi_6$, $D \leq \min\{\frac{a(1-\pi_{22})}{b\pi_{22}}, \frac{b\pi_{11}}{a(1-\pi_{11})}\}$,

where $$\pi_1 \equiv \frac{\frac{a}{a+b} - \pi_{22}}{\pi_{12} - \pi_{22}}, \quad \pi_2 \equiv \frac{\frac{a}{a+Db} - \pi_{22}}{\pi_{12} - \pi_{22}}$$

$$\pi_3 \equiv \frac{\frac{(D-1)b\pi_{12}}{a+b}}{\frac{(D-1)b\pi_{12}}{a+b} + \pi_{11} - \frac{a}{a+b}}, \quad \pi_4 \equiv \frac{\frac{(D-1)b\pi_{22}}{a+b}}{\frac{(D-1)b\pi_{22}}{a+b} + \pi_{12} - \frac{a}{a+b}}$$

$$\pi_5 \equiv \frac{b\pi_{12}}{a(1-\pi_{11}) + b\pi_{12}}, \quad \pi_6 = \frac{b\pi_{22}}{a(1-\pi_{12}) + b\pi_{22}}, \quad \tilde{D} \equiv \frac{b\pi_{12}}{a(1-\pi_{12})}.$$

Proof:

(I) This equilibrium has been discussed in detail.

(II) The situation is the same as (I) with the only exception being that n_{12}^1

= 2, which is optimal if $\pi \le \pi_1$.

(III) The payoff for $n^o_{11} = 2$ is $a(1 - \pi_{11})\pi + Db\pi_{12}(1 - \pi)$ while that from $n^1_{11} = n$ is $b\pi_{11}\pi + b\pi_{12}(1 - \pi)$. Thus $n^o_{11} = 2$ is optimal if $a(1 - \pi_{11})\pi + Db\pi_{12}(1 - \pi) \le b\pi_{11}\pi + b\pi_{12}(1 - \pi)$, or $\pi \ge \pi_3$. The payoff from $n^o_{12} = 2$ is $a(1 - \pi_{12})\pi + Db\pi_{22}(1 - \pi)$ while that of $n^1_{12} = k$ is $b\pi_{12}\pi + b\pi_{22}(1 - \pi)$ making $n^o_{12} = 2$ optimal if $\pi \ge \pi_4$. $n^o_{21} = 1$ is optimal if $D \ge \tilde{D}$ and so is $n^1_{21} = 1$. For $2(s_2)$ the equilibrium strategy $n^1_{22} = 2$ yields a payoff of $Db\pi_{12}\pi + Db\pi_{22}(1 - \pi)$ while that from $n^1_{22} = 1$ would yield $a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi)$. Then $n^1_{22} = 2$ would be optimal if $\pi \le \pi_2$.

(IV) The payoff from $n^1_{11} = 2$ is $Da(1-\pi_{11})\pi + b\pi_{12}(1-\pi)$ while that from $n^o_{11} = 2$ is $a(1-\pi_{11})\pi + Db\pi_{12}(1-\pi)$. This gives $\pi \le \pi_5$. Similarly, the condition $\pi \ge \pi_6$ can be derived.

Theorem 4.2: For $D \le \tilde{D}$, there are four possible pure strategy equilibria. These are

(I) $\{\infty, 1, 2, 1, \infty, 1, 1, 1\}$ for $\pi \ge \max\{\pi_7, \pi_8\}$,

(II) $\{\infty, 1, 1, 2, \infty, 1, 1, 1\}$ for $\pi \le \pi_7$,

(III) $\{\infty, 1, 2, 1, \infty, 1, 1, 2\}$ for $\pi_8 \ge \pi \ge \max\{\pi_9, \pi_6\}$,

(IV) $\{1, 2, 2, 1, \infty, 1, 2, 2\}$ for $\pi_5 \ge \pi$, $D \le \dfrac{a(1 - \pi_{22})}{b\pi_{22}}$,

$$\text{where } \pi_7 \equiv \frac{\frac{a}{a + b} - \pi_{22}}{\frac{(D - 1)a(1 - \pi_{12})}{a + b} + \frac{a}{a + b} - \pi_{22}},$$

$$\pi_8 \equiv \frac{\frac{a}{a + Db} - \pi_{22}}{\frac{(D^2 - 1)a(1 - \pi_{12})}{a + Db} + \frac{a}{a + Db} - \pi_{22}}$$

$$\pi_9 \equiv \frac{\dfrac{D^2a}{D^2a + b} - \pi_{12}}{\dfrac{D^2a}{D^2a + b} - \pi_{12} + \dfrac{(D - 1)a(1 - \pi_{11})}{D^2a + b}}$$

Proof : The remarks made for Theorem 4.1 hold except that now since $D \le \tilde{D}$, $n^o_{11} = \infty$ and $n^o_{21} = \infty$ are dominant strategies.

(I) For $n^o_{12} = 2$ the payoff is $a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi)$ while for $n^1_{12} = 2$ the payoff is $Da(1 - \pi_{12})\pi + b\pi_{22}(1 - \pi)$. Then the equilibrium strategy is optimal if $\pi \ge \pi_7$. Given that $2(s_2)$ believes that 1 is type s_2 if 1 says d_1, $n^o_{22} = 1$ is, also, optimal. The payoff for $n^1_{22} = 1$ is $a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi)$ while that from $n^1_{22} = 2$ is $D^2a(1 - \pi_{12})\pi + Db\pi_{22}(1 - \pi)$ so the equilibrium strategy is optimal if $\pi \ge \pi_8$.

(II) The payoff for $1(s_1)$ from playing $n^o_{11} = \infty$ is $a(1 - \pi_{11})\pi + D^2a(1 - \pi_{12})(1 - \pi)$ while that from $n^1_{11} = n$ is $Da(1 - \pi_{11})\pi + b\pi_{12}(1 - \pi)$. Then the equilibrium strategy is optimal if $\pi \ge \pi_9$. For $1(s_2)$ the payoff from $n^o_{12} = 2$ is $a(1 - \pi_{12})\pi + Db\pi_{22}(1 - \pi)$ while that from $n^1_{12} = 2$ is $Da(1 - \pi_{12})\pi + b\pi_{22}(1 - \pi)$. Then $n^o_{12} = 2$ is optimal if $\pi \ge \pi_6$. From above $n^1_{22} = 2$ is optimal if $\pi \le \pi_8$.

(III) The payoffs from $1(s_2)$'s strategies can be seen in (I). Now we will need $\pi \le \pi_5$. The rest of the strategies can be shown to be optimal from observing (I)-(II).

(IV) For $1(s_1)$ the condition required is $Da(1-\pi_{11})\pi + Da(1-\pi_{12})(1-\pi) \le a(1-\pi_{11})\pi + D^2a(1-\pi_{12})$. Using the condition $D \le \tilde{D}$ this reduces to $\pi \le \pi_5$.

Theorem 4.3: Let $\delta^{o2}_{11} = \delta^{o2*}_{11}$ and $\delta^{o2}_{12} = \delta^{o2*}_{12}$ be such that

$$\delta^{o2*}_{11}\pi\{a(1 - \pi_{12}) - Db\pi_{12}\} + \delta^{o2*}_{12}(1 - \pi)\{a(1 - \pi_{22}) - Db\pi_{22}\} = 0.$$

Also, let $\delta^{11}_{22} = \frac{\pi\{b\pi_{12} - a(1-\pi_{12})\} + (1-\pi)(1-D)b\pi_{22}}{(1-\pi)\{a(1-\pi_{22}) - Db\pi_{22}\}}$

$$= \frac{\pi\{b\pi_{11} - a(1 - \pi_{11})\} + (1-\pi)(1-D)b\pi_{12}}{(1 - \pi)\{a(1-\pi_{12}) - Db\pi_{12})}$$

Then, if the conditions,

$$(1 - \delta^{02*}_{11})\pi\{b\pi_{11} - Da(1 - \pi_{11})\} + (1 - \delta^{02*}_{12})(1 - \pi)\{b\pi_{12} - Da(1 - \pi_{12})\} \le 0,$$

and

$$(1 - \delta^{02*}_{11})\pi\{b\pi_{12} - Da(1 - \pi_{12})\} + (1 - \delta^{02*}_{12})(1 - \pi)\{b\pi_{22} - Da(1 - \pi_{22})\} \le 0$$

are met

$\delta^{02}_{11} = \delta^{02*}_{11}$, $\delta^{02}_{12} = \delta^{02*}_{12}$

$\delta^{01}_{21} = 1$, $\delta^{11}_{21} = 1$

$\delta^{01}_{22} = 1$, $\delta^{11}_{22} = \frac{\pi\{b\pi_{12} - a(1-\pi_{12})\} + (1-\pi)(1-D)b\pi_{22}}{(1-\pi)\{a(1-\pi_{22}) - Db\pi_{22}\}}$

constitutes a mixed strategy Nash equilibrium for $D \ge \tilde{D}$.

Proof: Note that $n^1_{21} = 1$ is a dominant strategy and both types of 1 would say d_1 if 2 said d_1 when 1 said d_0 in the previous period. Let $\delta^{02}_{11} = \delta^{02*}_{11}$, $\delta^{02}_{12} = \delta^{02*}_{12}$ and $\delta^{12*}_{11} = 1 - \delta^{02*}_{11}$, $\delta^{12*}_{12} = 1 - \delta^{02*}_{12}$. Thus $1(s_1)$ mixes between saying d_0 once ($n^0_{11} = 2$) and saying d_1 for one period ($n^1_{11} = 2$) and $1(s_2)$ mixes between saying d_0 once ($n^0_{12} = 2$) and saying d_1 once ($n^1_{12} = 2$). Let $2(s_2)$ play the strategy $\delta^{11}_{22} = \delta^{11*}_{22}$, $\delta^{12}_{22} = \delta^{12*}_{22} = 1 - \delta^{11*}_{22}$, so that $2(s_2)$ mixes between $n^1_{22} = 1$ and $n^1_{22} = 2$. The payoff for $1(s_1)$ from $n^0_{11} = 2$ is $a(1 - \pi_{11})\pi + a(1 - \pi_{12})(1 - \pi)\delta^{11*}_{22} + Db\pi_{12}(1 - \pi)(1 - \delta^{11*}_{22})$ and that from $n^1_{11} = 2$ is $b\pi_{11}\pi + b\pi_{12}(1 - \pi)$. Since $1(s_1)$ must be indifferent between his pure strategies that means

$$\delta^{11*}_{22} = \frac{\pi\{b\pi_{11} - a(1 - \pi_{11})\} + (1 - \pi)(1 - D)b\pi_{12}}{(1 - \pi)\{a(1 - \pi_{12}) - Db\pi_{12}\}}$$

Similarly $1(s_2)$ must be indifferent between his pure strategies. The payoff from $n^{o}_{12} = 1$ is $a(1 - \pi_{12})\pi + a(1 - \pi_{22})(1 - \pi)\delta^{11*}_{22} + Db\pi_{22}(1 - \pi)(1 - \delta^{11*}_{22})$ and that from $b\pi_{12}\pi + b\pi_{22}(1 - \pi)$. Then

$$\delta^{11*}_{22} = \frac{\pi\{b\pi_{12} - a(1-\pi_{12})\} + (1-\pi)(1-D)b\pi_{22}}{(1-\pi)\{a(1-\pi_{22}) - Db\pi_{22}\}}$$

For 2 of both types $\mu(d_o) = \dfrac{\delta^{02*}_{11}\pi}{\delta^{02*}_{11}\pi + \delta^{02*}_{12}(1 - \pi)}$

and $\mu(d_1) = \dfrac{(1 - \delta^{02*}_{11})\pi}{(1 - \delta^{02*}_{11})\pi + (1 - \delta^{02*}_{12})(1 - \pi)}$

The payoffs for $2(s_2)$ from $n^{1}_{22} = 1$ and $n^{1}_{22} = 2$ are

$$a(1 - \pi_{12}) \frac{\delta^{02*}_{11}\pi}{\delta^{02*}_{11}\pi + \delta^{02*}_{12}(1 - \pi)} + a(1 - \pi_{22}) \frac{\delta^{02*}_{11}(1 - \pi)}{\delta^{02*}_{11}\pi + \delta^{02*}_{12}(1 - \pi)}$$

and $Db\pi_{12} \dfrac{\delta^{02*}_{11}\pi}{\delta^{02*}_{11}\pi + \delta^{02*}_{12}(1 - \pi)} + Db\pi_{22} \dfrac{\delta^{02*}_{11}(1 - \pi)}{\delta^{02*}_{11}\pi + \delta^{02*}_{12}(1 - \pi)}$

$2(s_2)$ should be indifferent between these strategies so that

$$\delta^{02*}_{11}\ \pi\{a(1 - \pi_{12}) - Db\pi_{12}\} + \delta^{02*}_{12}\ (1 - \pi)\{a(1 - \pi_{22}) - Db\pi_{22}\} = 0.$$

It remains to be shown that $n^{o}_{21} = 1$ and $n^{o}_{22} = 1$ are optimal. The payoff from $n^{o}_{21} = 1$ is

$$\frac{(1-\delta_{11}^{02*})\pi}{(1-\delta_{11}^{02*})\pi + (1-\delta_{12}^{02*})(1-\pi)} b\pi_{11} + \frac{(1-\delta_{11}^{02*})(1-\pi)}{(1-\delta_{11}^{02*})\pi + (1-\delta_{12}^{02*})(1-\pi)} b\pi_{12}$$

The payoff from $n_{21}^{o} = 2$ is

$$\frac{(1-\delta_{11}^{02*})\pi}{(1-\delta_{11}^{02*})\pi + (1-\delta_{12}^{02*})(1-\pi)} Da(1-\pi_{11})$$

$$+ \frac{(1-\delta_{11}^{02*})(1-\pi)}{(1-\delta_{11}^{02*})\pi + (1-\delta_{12}^{02*})(1-\pi)} Da(1-\pi_{12}).$$ Then

$$(1-\delta_{11}^{02*})\pi\{b\pi_{11} - Da(1-\pi_{11})\} + (1-\delta_{12}^{02*})(1-\pi)\{b\pi_{12} - Da(1-\pi_{12})\} \leq 0.$$

Similarly for $n_{22}^{o} = 1$ to be optimal it must be the case that

$$(1-\delta_{11}^{02*})\pi\{b\pi_{12} - Da(1-\pi_{12})\} + (1-\delta_{12}^{02*})(1-\pi)\{b\pi_{22} - Da(1-\pi_{22})\} \leq 0.$$

Corollary: If $D \geq \tilde{D}$ and $\min\{\pi_1, \pi_2\} \geq \pi \geq \max\{\pi_3, \pi_4\}$

then $\delta_{11}^{02} = 1$, $\delta_{12}^{02} = (1 - \delta_{12}^{12}) = \dfrac{\pi}{1-\pi} \dfrac{\pi_{12} - \frac{a}{a+Db}}{\frac{a}{a+Db} - \pi_{22}}$

$\delta_{21}^{01} = 1$, $\delta_{21}^{11} = 1$

$\delta_{22}^{01} = 1$, $\delta_{22}^{11} = \dfrac{\pi\{b\pi_{12} - a(1-\pi_{12})\} + (1-\pi)(1-D)b\pi_{22}}{(1-\pi)\{a(1-\pi_{22}) - Db\pi_{22}\}}$

is a mixed strategy Nash equilibrium.

Proof: From theorem 3.1 if $\delta^{11}_{22} = 0$, then $\delta^{02*}_{11} = 1$ if $\pi \geq \pi_3$. The rest of the results follow from substitution of this result.

Theorem 4.4: If $D \leq \tilde{D}$ and $\min\{\pi_7, \pi_8\} \geq \pi \geq \max\{\pi_6, \pi_9\}$ then

$$\delta^{0\infty}_{11} = 1,\ \delta^{02}_{12} = \frac{\pi}{1-\pi}\frac{(D^2-1)a(1-\pi_{12})}{a(1-\pi_{22}) - Db\pi_{22}}$$

$$\delta^{0\infty}_{21} = 1,\ \delta^{11}_{21} = 1$$

$$\delta^{01}_{22} = 1,\ \delta^{11}_{22} = \frac{(D-1)\{\pi a(1-\pi_{12}) - (1-\pi)b\pi_{22}\}}{(1-\pi)\{a(1-\pi_{22}) - Db\pi_{22}\}}$$

is a mixed strategy Nash equilibrium.

Proof: For $1(s_1)$, $n^0_{11} = \infty$ has a payoff of $a(1-\pi_{11})\pi + a(1-\pi_{12})(1-\pi)\delta^{11}_{22} + Da(1-\pi_{12})(1-\pi)(1-\delta^{11}_{22})$ while that from $n^1_{11} = 2$ is $Da(1-\pi_{11})\pi + b\pi_{12}(1-\pi)$. For $1(s_2)$ the payoffs for $n^0_{11} = 2$ and $n^1_{11} = 2$ are $a(1-\pi_{12})\pi + a(1-\pi_{22})(1-\pi)\delta^{11}_{22} + Db\pi_{22}(1-\pi)(1-\delta^{11}_{22})$ and $Da(1-\pi_{12})\pi + b\pi_{22}(1-\pi)$. For $2(s_2)$ the payoffs from $n^1_{22} = 1$ and $n^1_{22} = 2$ are

$$a(1-\pi_{12})\frac{\pi}{\pi + \delta^{02*}_{12}(1-\pi)} + a(1-\pi_{22})\frac{\delta^{02*}_{12}(1-\pi)}{\pi + \delta^{02*}_{12}(1-\pi)}$$

and $$D^2a(1-\pi_{12})\frac{\pi}{\pi + \delta^{02*}_{12}(1-\pi)} + Db\pi_{22}\frac{\delta^{02*}_{12}(1-\pi)}{\pi + \delta^{02*}_{12}(1-\pi)}$$

The required results can now be derived using steps used in the proof of

Theorem 4.3 and the payoffs given above.

Theorem 5.1: Let $D \geq \max(\bar{D}, \hat{D})$, then there are four possible pure strategy equilibria. These are

(I) {2, 1, 2, 1, 1, 1, 1, 1} for $\pi \geq \max\{\beta_{11}, \beta_{21}\}$,

(II) {2, 1, 1, 2, 1, 1, 1, 1} for $\pi \leq \beta_{11}$,

(III) {2, 1, 2, 1, 1, 1, 1, 2} for $\beta_{21} \geq \pi \geq \max\{\beta_{12}, \beta_{13}\}$,

(IV) {1, 2, 2, 1, 2, 1, 1, 2} for $\beta_{14} \geq \pi \geq \beta_{15}$, $D \leq \min\{\frac{a_2(1-\pi_{22})}{b\pi_{22}}, \frac{b\pi_{11}}{a_2(1-\pi_{11})}\}$,

where $\beta_{11} \equiv \dfrac{\frac{a_1}{a_1 + b} - \pi_{22}}{\pi_{12} - \pi_{22}}$, $\beta_{21} \equiv \dfrac{\frac{a_2}{a_2 + Db} - \pi_{22}}{\pi_{12} - \pi_{22}}$,

$$\beta_{12} \equiv \frac{\frac{(D-1)b\pi_{12}}{a_1 + b}}{\frac{(D-1)b\pi_{12}}{a_1 + b} + \pi_{11} - \frac{a_1}{a_1 + b}}, \quad \beta_{13} \equiv \frac{\frac{(D-1)b\pi_{22}}{a_1 + b}}{\frac{(D-1)b\pi_{22}}{a_1 + b} + \pi_{12} - \frac{a_1}{a_1 + b}}$$

$$\beta_{14} \equiv \frac{b\pi_{12}}{a_1(1-\pi_{11}) + b\pi_{12}}, \quad \beta_{15} \equiv \frac{b\pi_{22}}{a_1(1-\pi_{12}) + b\pi_{22}}$$

$$\hat{D} \equiv \frac{a_2(1-\pi_{12})}{b\pi_{12}}, \quad \bar{D} \equiv \frac{b\pi_{12}}{a_1(1-\pi_{12})}$$

Theorem 5.2: Let $\bar{D} \geq D \geq \hat{D}$, then there are four possible pure strategy equilibria. These are

(I) {∞, 1, 2, 1, 1, 1, 1, 1} for $\pi \geq \max\{\beta_{11}, \beta_{22}\}$,

(II) {∞, 1, 1, 2, 1, 1, 1, 1} for $\pi \leq \beta_{11}$,

(III) {∞, 1, 2, 1, 1, 1, 1, 2} for $\beta_{22} \geq \pi \geq \max\{\beta_{13}, \beta_{16}\}$,

(IV) {1, 2, 2, 1, 2, 1, 1, 2} for $\beta_{17} \geq \pi \geq \beta_{15}$, $D \leq$

$\min\{ \frac{a_2(1 - \pi_{22})}{b\pi_{22}}, \frac{b\pi_{11}}{a_2(1 - \pi_{11})}\}.$

$$\text{where } \beta_{16} \equiv \frac{\frac{D^2 a_1}{D^2 a_1 + b} - \pi_{12}}{\frac{D^2 a_1}{D^2 a_1 + b} - \pi_{12} + \frac{a_1 + b}{D^2 a_1 + b}\pi_{11} - \frac{a_1}{D^2 a_1 + b}},$$

$$\beta_{17} \equiv \frac{\frac{D^2 a_1}{D^2 a_1 + b} - \pi_{12}}{\frac{D^2 a_1}{D^2 a_1 + b} - \pi_{12} + \frac{(D - 1)a_1(1 - \pi_{11})}{D^2 a_1 + b}}.$$

$$\beta_{22} \equiv \frac{\frac{a_2}{a_2 + Db} - \pi_{22}}{\frac{(D^2 - 1)a_2(1 - \pi_{12})}{a_2 + Db} + \frac{a_2}{a_2 + Db} - \pi_{22}}$$

Theorem 5.3: If $\hat{D} \geq D \geq \bar{D}$, there are four possible Nash equilibrium. These are

(I) {2, 1, 2, 1, 1, 1, 1, ∞} for $\pi \geq \max\{\beta_{12}, \beta_{13}, \beta_{24}\}$,

(II) {2, 1, 1, 2, 1, 1, 1, ∞} for $\beta_{13} \geq \pi \geq \beta_{12}$,

(III) {1, 2, 1, 2, 1, 1, 1, ∞} for $\pi \leq \min\{\beta_{12}, \beta_{13}, \beta_{23}\}$,

(IV) {1, 2, 1, 2, 2, 1, 1, ∞} for $\beta_{23} \leq \pi \leq \min\{\beta_{14}, \beta_{15}\}$,

$$\text{where } \beta_{23} \equiv \frac{\frac{Da_2}{Da_2 + b} - \pi_{12}}{\pi_{11} - \pi_{12}}, \quad \beta_{24} \equiv \frac{\frac{a_2}{a_2 + Db} - \pi_{12}}{\pi_{11} - \pi_{12}}$$

Theorem 5.4: If $D \geq \max\{\bar{D}, \hat{D}\}$ and player 2 starts the game, there are four Nash equilibria. These are

(I) {1, 2, 1, 2, 1, 1, 1, 1}.

(II) {2, 1, 1, 2, 1, 1, 1, 1}.

(III) {1, 2, 1, 2, 2, 1, 1, 1}.

(IV) {1, 2, 2, 1, 2, 1, 1, 2}.

Theorem 5.5: If $D \le \min\{\bar{D},\hat{D}\}$ there are seven pure strategy equilibria. They are

(I) $\{\infty, 1, 2, 1, 1, 1, 1, 1\}$ for $\pi \ge \max\{\beta_{11},\beta_{22},\beta_{25}\}$,

(II) $\{\infty, 1, 1, 2, 1, 1, 1, 1\}$ for $\pi \le \beta_{11}$,

(III) $\{\infty, 1, 2, 1, 1, 1, 1, 2\}$ for $\beta_{25} \ge \pi \ge \max\{\beta_{13},\beta_{16},\beta_{22}\}$,

(IV) $\{2, 1, 2, 1, 1, 1, 1, \infty\}$ for $\pi \ge \max\{\beta_{12},\beta_{13},\beta_{22}\}$

(V) $\{2, 1, 1, 2, 1, 1, 1, \infty\}$ for $\beta_{13} \ge \pi \ge \beta_{12}$,

(VI) $\{1, 2, 1, 2, 1, 1, 1, \infty\}$ for $\pi \le \min\{\beta_{12}, \beta_{13}, \beta_{23}\}$,

(VII) $\{1, 2, 1, 2, 2, 1, 1, \infty\}$ for $\beta_{23} \le \pi \le \min\{\beta_{14}, \beta_{15}\}$,

$$\text{where } \beta_{25} \equiv \frac{\dfrac{a_2}{a_2 + Db} - \pi_{12}}{\dfrac{(D^2 - 1)a_2(1 - \pi_{11})}{a_2 + Db} + \dfrac{a_2}{a_2 + Db} - \pi_{12}}$$

Proof: As stated earlier the proof is similar to that of Theorem 4.1 and 4.2. One point to note is that since $D \le \min\{\bar{D},\hat{D}\}$, if the event is (s_1,s_2) then both 1 and 2 should insist on their particular preferences. Thus there can be two different Nash equilibria; one in which 1 concedes and another one where 2 concedes once the player's types are known to each other. The proof is then straightforward.

Proof of Proposition 5.7: (a) Suppose $\sigma_2^{k+1}(d_0|(h^k,d_0),\mu(s_1|h^{k+1})) = 0$, then $u_1(\sigma_1^k(d_0|\mu(h^k)) = u_1(\sigma_1^{k+2}(d_0|\mu(h^{k+2}))$ since both types of player 2 say d_1 and so no new information is available about player 2's type. Then if

$u_1(\sigma_1^k(d_0|\mu(h^k)) = 1$ is optimal it implies that $u_1(\sigma_1^{k+2}(d_0|\mu(h^{k+2})) = 1$ is also optimal. Similarly it can be shown that in that if $1(s_2)$ is going to say d_0 with probability one in stage k + 2 then if $2(s_1)$ says d_1 in stage k + 1 he should say d_1 in stage k + 3. Then by induction it can be shown that player 1 and 2 will alternate between d_0 and d_1. (b) can be shown in a similar manner. For (c) note that for any stage where $1(s_2)$ and $2(s_1)$ play d_0 and d_1 the payoffs have to be the same. This is a feature of mixed strategies. However this conflicts with another feature of mixed strategies that the probabilities of the mixtures should add up to one. It is easier to show (c) using mixed strategies. Suppose $\delta_{12}^{1n} > 0$. Then $S(n_{21}^1 = n) = S(n_{21}^1 = n + 1)$. Simplifying this condition we get

$$\delta_{12}^{1n} = \frac{(D - 1)}{Da_2(1 - \pi_{12}) - b\pi_{12}} \left[a_2(1 - \pi_{12})(1 - \sum_{k=1}^{n-1} \delta_{12}^{1k}) + \frac{a_2(1 - \pi_{11})}{(1 - \pi)} \right]$$

$$> \frac{(D - 1)}{Da_2(1 - \pi_{12}) - b\pi_{12}} \frac{a_2(1 - \pi_{11})}{(1 - \pi)}$$

which contradicts $\sum_{n=1}^{\infty} \delta_{12}^{1n} = 1$.

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