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Abstract:
We consider a revenue sharing contract in a supply chain, under price dependent demand. The demand is random and follows a multiplicative model. We show that the retail price that maximizes expected retailer profits, is higher than the price that would maximize profits if there was no demand uncertainty. We then show that if the wholesale price is sufficiently low, there is a positive revenue sharing fraction that the manufacturer would prefer. This preference is in comparison to not entering into a revenue sharing contract at all.

Keywords: Supply Chain Management, Revenue Sharing Contract, Price Dependent Demand

1. Introduction and Literature Review

The phenomenon of double marginalization in a supply chain is well accepted. Each party makes decisions concerning only their portion of the supply chain margin. The margin of the retailer is smaller than the supply chain margin. The supply margin is one that would be in place if there was a single decision maker for the supply chain consisting of the manufacturer and the retailer. Also the overstocking cost of the retailer is higher than the overstocking cost of the supply chain. Because of these two reasons the retailer stocks fewer volume of product than is optimal for the supply chain. This reduces supply chain profits.

Buyback contracts, Revenue Sharing contracts and Quantity Flexibility contracts are some of the contracts that have been proposed to overcome the problem of double marginalization. These contracts overcome the problem by offering incentives to the retailer to increase the order quantity. In a buyback contract, the manufacturer specifies a wholesale price and a buyback price at which to purchase unsold units at the end of the season. This increases the salvage value per unit for the retailer, who then orders more. In a revenue sharing contract, the manufacturer shares a fraction of the revenue of the retailer and offers the retailer a low wholesale price. The low wholesale price reduces the overstocking risk for the retailer, who then orders more. In the revenue sharing contract, the manufacturer shares the risk (of having unsold units) of the retailer. If the customer sales are high, both the manufacturer and retailer benefit. If sales are low, both lose out.

It is well documented (Cachon (2001), Chopra and Meindl (2004)), that revenue sharing contracts and buyback contracts increase profits for both parties resulting in a win-win situation, if properly implemented. But do these same benefits carry over if demand is price dependent. Emmons and Gilbert (1998), have shown in a model of price dependent demand, that buyback policies would benefit both the retailer and the manufacturer. The authors show that given that the wholesale price is in a certain range, the manufacturer chooses a positive buyback price over not offering a buyback price.

In the present paper, we use the same model of price dependent demand as Emmons and Gilbert (1998). We consider the revenue sharing contract in this setting. We show that the retail price that maximizes the expected retailer profits, is higher than the price that would maximize profits if there was no demand uncertainty. We then show that if the wholesale price is sufficiently low, there is a positive revenue sharing fraction that the manufacturer would prefer. This preference is in comparison to not entering into a revenue sharing contract at all.

Mortimer (2000) considers an econometric study of revenue sharing contracts in the video rental industry. Gerchak, Cho and Ray (2001) consider a decision oriented model of shelf space management in the video rental industry. Expositions of the revenue sharing contract can be found in Cachon (2001) and Chopra and Meindl (2004). Cachon and Lariviere (2005) consider revenue sharing contracts in a general supply chain model in which demand can be deterministic or stochastic. They consider the standard newsboy model as well as the price setting newsboy. Also revenue sharing contracts are compared to other supply chain contracts. There are also models of price dependent demand in which a pricing decision and an inventory decision is jointly considered. Lau and Lau (1988) consider the newsboy problem with price dependent demand distributions. Gerchak and Parlar (1987) consider a model in which a joint decision is made on inventory levels and marketing effort. Finally, Petruzzi and Dada (1999), have a review article on pricing and the newsvendor problem.

This paper is organized as follows. In Section 2, we formulate the model and consider the retailer's decision. In Section 3, we consider the manufacturer's decision. Finally, Conclusions are presented in Section 4.

2. Model Formulation: The Retailer's Decision

The manufacturer charges a wholesale price c for the product sold at the start of the selling season. The manufacturer also shares a fraction of the revenues f of the retailer. A production cost,v, is incurred by the manufacturer per unit. Given the wholesale price,c, and the revenue share fraction, f, the retailer chooses the order quantity, x, and the price, p, at which to sell the product to the consumer.

The retailer faces a price dependent demand in accordance with a multiplicative demand model. In particular, the demand,
$X(p,Y) = D(p)*Y$
Here Y is a positive random variable, with $E[Y] = 1$ and with cumulative distribution F(.). The function D(p) is linear and decreasing in the price, p. We assume $D(p) = b(p-k)$, with $b<0$ and $k>0$ are constants. Then, we have that,
$E[X(p,Y)] = D(p)*E[Y] = D(p)$

To analyze the problem, we first assume that the wholesale price, c, and the revenue share fraction, f, of the manufacturer are fixed. Given these two parameters, we determine the retailer's decisions. That is, we determine the order quantity, x, and the price, p, that the retailer would offer to the consumer. Once we determine the retailer's decision for a fixed c and f, we then analyze the manufacturer's decision.

Let $\pi_R(p,x;c,f)$ denote the retailer's expected profit, given the manufacturer's decisions, c and f. Then,
$\pi_R(p,x;c,f) = (1-f)pE[\min\{x,X\}] - cx$
The optimal order quantity for the retailer, x^*, which maximizes $\pi_R(p,x;c,f)$, is given by,
$x^* = D(p)F^{-1}(\ 1-c/((1-f)p)\ )$
We then have the following result, similar to Emmons and Gilbert (1998).

Proposition 1: The retail price p^* that maximizes the expected retailer profits, $\pi_R(p,x;c,f)$, is higher than the price that would maximize profits if there was no demand uncertainty. In the case if no demand uncertainty, $X(p) = D(p)$.
Proof: If there was no demand uncertainty, the retailer's profit, $\pi_R{}^d\,(p;c,f)$, would be given by,

$\pi_R{}^d(p;c,f) = [(1-f)p- c]D(p)$
with $x=X(p)=D(p)$.
We have that pD(p) is concave in p, given the assumption that demand is linear and decreasing in price. Therefore, the optimal price, p_d, in the deterministic case, satisfies the following equation,
$[(1-f)\ p_d - c][d/dp_d\ \{D(p_d)\}] + (1-f)*D(p_d) = 0$

Let $p^{RS} = (1-f)p$. We have that,
$p^{RS}[\min\{x,X\}] - cx$ can be written as,
$(p^{RS} - c)X + (p^{RS} - c)(x-X) - p^{RS}*(x-X)^+$
Substituting, $(x-X) = (x-X)^+ - (X-x)^+$ in the above and taking expectations, we have that,
$\pi_R(p,x;c,f) = (1-f)pE[\min\{x,X\}] - cx$
$= (p^{RS} - c)E[X] - (p^{RS} - c)E[(X-x)^+] - c*E[(x-X)^+]$
Writing $y = x/D(p)$, we have
$\pi_R(p,y;c,f) = (p^{RS} - c)D(p) - (p^{RS} - c)D(p)E[(Y-y)^+] - cD(p)*E[(y-Y)^+]$
Taking the derivative of $\pi_R(p,y;c,f)$ with respect to p, we have,
$d/dp\{\pi_R(p,y;c,f)\} = \{\ [(1-f)\ p - c][d/dp\ \{D(p)\}] + (1-f)*D(p)\ \}*(1- E[(Y-y)^+])$
$-\ c[d/dp\ \{D(p)\}]*E[(y-Y)^+]$ (1)
We then have the following,
$E[(Y-y)^+] < E[Y] - y[1-F(y)] < 1$ (as $E[Y] = 1$)
and $d/dp\ \{D(p)\} < 0$. Also, $E[(y-Y)^+] > 0$.

Since, $\pi_R{}^d(p;c,f) = (\ (1-f)p - c)D(p)$ is concave, we have that,
$[(1-f)\ p_d - c][d/dp_d\ \{D(p_d)\}] + (1-f)*D(p_d) >= 0$
for all $p <= p_d$.

Substituting the above relations in (1), we have that,
$d/dp\{\pi_R(p,y;c,f)\} > 0$ for all $p <= p_d$.
Thus $\pi_R(p,y;c,f)$ must achieve its maximum at some point higher than p_d.

We now find out the retailer's optimal profit at a price p, as determined by $x^*(p)$. To make the expression tractable, we now assume that Y has a uniform distribution between [0,2]. The same simplifying assumption has also been made in Emmons and Gilbert (1998).

We have,
$\pi_R(p,y^*;c,f) = (p^{RS} - c)D(p) - (p^{RS} - c)D(p)E[(Y- y^*)^+] - cD(p)*E[(y^*-Y)^+]$

Here Y is U[0,2] and $y^* = 2*((p^{RS} - c)/p^{RS})$, where $p^{RS} = (1-f)*p$

We then have,
$\pi_R(p,y^*;c,f) = \pi_R(p;c,f) = D(p)*\ ((p^{RS} - c)^2/p^{RS})$,
or
$\pi_R(p;c,f) = (b(p-k))*\ (((1-f)p- c)^2/(1-f)p)$, with $b<0$ and $k>0$ as defined previously.

By setting the derivative of $\pi_R(p;c,f)$, with respect to p, equal to zero, the retailer's profit maximizing price is given by,
$p^* = k/4 + sqrt(k^2/16 + ck/(2(1-f)))$ (2)
and
$x^* = D(p^*)*2*((p^{RS} - c)/p^{RS})$ (3)
or
$x^* = 2(b(p^*-k))* (((1-f)\, p^* - c)/(1-f)\, p^*)$

We next consider the manufacturer's decision.

3. The Manufacturer's Decision

The manufacturer charges a wholesale price c for the product sold at the start of the selling season. The manufacturer also shares a fraction of the revenues f of the retailer. A production cost,v, is incurred by the manufacturer per unit. Given the wholesale price, c, and the revenue share fraction, f, the retailer chooses the order quantity, x^*, and the price, p^*, at which to sell the product to the consumer. The optimal price and the optimal order quantity of the retailer have been defined above using equations (2) and (3). This has been done with the assumption of a uniform distribution for the random variable Y.

We now consider the manufacturer's decision. The manufacturer's profit $\pi_M(c,f)$ is given by,
$\pi_M(c,f) = (c-v)* x^*(c,f) + f* p^*(c,f)*E\min\{x^*(c,f),X\}$ (4)

where $p^*(c,f)$ and $x^*(c,f)$ are as in (2) and (3).

We have that,
$E\min\{x^*,X\} = x^* - (x^*)^2/(4*D(p^*))$ (5)

Then, using (4) and (5),
$\pi_M(c,f) = (c- v + fp^*)* x^* - (fp^*(x^*)^2)/(4*D(p^*))$ (6)

Proposition 2: There is a threshold, $c_T > v$, such that for any $v < c < c_T$, there exists a revenue share fraction f at which the manufacturer receives a higher profit than when f=0.
Proof: We need to show that $d/df(\pi_M(c,f))$ evaluated at f=0 is positive. From (6), we have that,
$d/df\,(\pi_M(c,f=0)) = (c- v)dx^*/df + p^*x^* - (p^*(x^*)^2)/(4*b(p^*-k))$ (7)
From (3), we have,
$x^*(c,f) = 2b(p^*-k) - (2bc(p^*-k))/(1-f)p^*)$ (8)

We then have from (8),
$d/df\,(x^*(c,f=0)) = 2b*d/df(p^*(c,f=0)) - 2bc/((p^*)^2)[k*d/df(p^*(c,f=0)) + p^*(p^*-k)]$ (9)

Using the expression of p^* from (2), we have, $d/df(p^*(c,f))$ evaluated at f=0 is given by,
$$d/df(p^*(c,f=0)) = ck/(4*(p^* - k/4)) \qquad (10)$$

We have that $d/df(p^*(c,f=0)) > 0$

Substituting the expression for $d/df(p^*(c,f=0))$ from (10) into (9), we have,
$$d/df\,(x^*(c,f=0)) = 2bck/(4*(p^* - k/4)) - 2bck/((p^*)^2)*d/df\,(p^*(c,f=0)) - 2bc + 2bck/p^* \qquad (11)$$
Out of the four terms in the RHS of (11), the second and third terms are positive (since $b< 0$). We have that,
$$d/df\,(x^*(c,f=0)) > bck/(2*(p^* - k/4)) + 2bck/(p^* - k/4) = 5bck/(2*(p^* - k/4))$$
So,
$$(c-v)d/df(x^*(c,f=0)) > 5bck(c-v)/(2*(p^* - k/4)) \qquad (12)$$
As $p^* > k/4$ and $b<0$, the RHS of (12) is negative.

The inequality in (12) is for the first term of (7). The second and third terms of (7) can be written as,
$$p^*x^* - (p^*(x^*)^2)/(4*b(p^*-k)) = [p^*x^*/(p^*-k)]*[p^*-k - x^*/4b] \quad (13)$$

Evaluating $x^*(c,f)$ at f=0 and using (3), we have,
$$p^*x^*/(p^*-k) = 2b(p^* - c) \qquad (14)$$
and
$$x^*/4b = ((p^*-k)/2)*(1- c/p^*) \qquad (15)$$

Using (13), (14) and (15), the second and third terms of (7), can be written as,
$$p^*x^* - (p^*(x^*)^2)/(4*b(p^*-k)) = 2b(p^* - c)*(p^*-k)[1- 1/2*(1- c/p^*)] \qquad (16)$$

We have $p^* > c$, $p^* < k$ and $b<0$. So the RHS of (16) is positive. Using (7), (12) and (16), to show,
$d/df\,(\pi_M(c,f=0)) > 0$ it would be sufficient to show that,
$$5bck(c-v)/(2*(p^* - k/4)) + 2b(p^* - c)*(p^*-k)[1- 1/2*(1- c/p^*)] > 0 \quad (17)$$

The above equation (17) can be simplified to,
$$(4/5k)*(p^* - c)*(k-p^*)*[1- 1/2*(1- c/p^*)]*(p^* - k/4) > c(c-v) \qquad (18)$$

Choosing $c = v + \varepsilon$, for $\varepsilon>0$ and arbitrarily small, we see that (18) reduces to,
$$(4/5k)*(p^* - v - \varepsilon)*(k-p^*)*[1- 1/2*(1- (v + \varepsilon)/p^*)]*(p^* - k/4) > \varepsilon(v + \varepsilon) \ (19)$$

We see that if we choose ε small enough (19) would hold. We have thus shown that, $d/df\,(\pi_M(c,f=0)) > 0$.

Proposition 2 shows that if the wholesale price is low and sufficiently close to the manufacturing price, there is a positive revenue sharing fraction that the manufacturer would prefer, as it increases profits. This result is not clear apriori. If the revenue sharing fraction, f, increases the manufacturer receives a higher revenue for each unit sold at the retailer. Thus an increase in f, increases profits for the manufacturer. But as f increases, the retailer orders fewer units from the manufacturer. So the initial margin on selling the units, (c-v), is made on fewer units. This decreases manufacturer profits.

In the next section we present our Conclusions.

4. Conclusions

We consider a revenue sharing contract under price dependent demand. The demand is random and follows a multiplicative model. We show that the retail price that maximizes the expected retailer profits, is higher than the price that would maximize profits if there was no demand uncertainty. We then show that if the wholesale price is sufficiently low, there is a positive revenue sharing fraction that the manufacturer would prefer, as it increases profits. This preference is in comparison to not entering into a revenue sharing contract at all.

Similar results have been obtained for two of the contracts under price dependent demand. It remains to be seen if the results also hold for other contracts. We are in the process of working on a similar demand model, but this time with the quantity flexibility contract.

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