WP- 45

PROJECT RESOURCE SCHEDULE AND OPTIMAL INVENTORY POLICY

by

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1. Introduction

Project Management (PERT/CPM) is concerned with the implementation of activities of a project based on constraints imposed by time and resources. Present computer packages for Project Management (PM) are quite good in generating such activity schedule for the project manager to implement the project.

Generally resource analysis is quite complex. Even though elegant Linear Programming (LP) modelling is possible for the resource analysis, the size of the LP, even for a moderate size network will be so large that the present-day fastest computer may not be able to solve it in reasonable time. Hence, heuristic approach is the accepted method of solving such problems. Resources may be manpower type (Rate type) or consumable type (Total type). Manpower has to be utilized during the period of availability, otherwise it is waste. On the other hand, consumable type of resources like cement, steel, etc., can be carried forward to the next period, if it is not used in the present period. PM packages are good in deciding the optimal schedule and corresponding resource profile i.e. requirement of a specific resource for each period.

Even though resource profile itself is one of the important outputs of PM, it is not completely helpful for the Project Manager. He has to decide on the optimal staffing

pattern for the Rate Resource and the optimal inventory policy for the Total Resource.

This paper is concerned with the strategy of inventory management for the Total Resource. A number of inventory models are available in the Operations Research (OR) literature [6]. Wagner-Whitin Model [1,2] is relevant for the present problem.

The paper briefly describes the concept of resource profile with an example in Section 2 and the selected inventory model in Section 3. Section 4 shows the integration of PM package and the Inventory Model so that the Project Manager may optimally stock the required resource and implement the project. The last section draws important conclusions and indicates further research topics.

2. Resource Profile of a Project

Project Management consists of following analysis :

* Time Analysis (TA)
* Resource Analysis (RA)
* Cost Analysis (CA)

Time Analysis is concerned with finding the critical path and completion time of the project. One can get a schedule by the Earliest Start or by the Latest Start.

Resource Analysis considers the requirement of resources for each activity and Total Availability of the Resource to

the project. Some activities of the schedule obtained after TA may require resource level which may not be available. These activities will be pushed till the required quantity of those resources are available. This may delay the project. If the project is not to be delayed, then the project manager will know the additional resource quantity to be made available. Requirement of a resource for the complete duration of the project is called "Resource Profile".

Cost Analysis is concerned with the aggregation of all the costs of usage of resources period by period for the complete project. This is termed as Direct Cost. If Indirect Cost is known then the addition of these two costs will lead to the total cost of the project period by period and the optimal duration of the project.

2.1 Maintenance of Electrical Equipment - Example :

A small example of electrical maintenance is considered for illustration. Fig. 1 shows the list of activities, their durations in days and result of Time Analysis. The project takes 19 days (assuming all are working days). It starts on 1 Sept 1992 and ends on 19 Sept 1992.

The Resources are :

Mechanics

Technicians

Oil-Grade-1A

power-unit Plan 19-09-92

ACTIVITY VIEW	Estimate	Start	End
- 1:disassemble power unit	[3ed]	01-09-92	03-09-92
- 2:test & repair instruments	[9ed]	01-09-92	09-09-92
- 3:clean main frame	[2ed]	01-09-92	02-09-92
- 4:procure new assembly	[1ed]	01-09-92	01-09-92
- 5:pull old assembly	[2ed]	01-09-92	02-09-92
- 6:clean, inspect & repair power unit	[4ed]	04-09-92	07-09-92
- 7:calibrate instruments	[8ed]	10-09-92	17-09-92
- 8:inspect & repair	[1ed]	03-09-92	03-09-92
- 9:change tangs	[1ed]	02-09-92	02-09-92
- 10:INSTALL NEW ASSEMBLY	[2ed]	03-09-92	04-09-92
- 11:assemble & test	[5ed]	08-09-92	12-09-92
- 12:check fittings	[3ed]	04-09-92	06-09-92
- 13:retrofit check	[1ed]	05-09-92	05-09-92
- 14:safety inspection	[1ed]	18-09-92	18-09-92
- 15:system check	[2ed]	18-09-92	19-09-92

3:clean main frame [2ed]
Planned Start:01-09-92 End:02-09-92 Slack:11d Total:R1160
F2/HELP F10/MENU

FIG 1: ACTIVITY LIST & TIME ANALYSIS

10 Mechanics and 5 Technicians are available. They work 8 hours/day and cost Rs.5/hour and Rs.10/hour respectively. Oil-Grade-1A is the consumable resource available at Rs.100/litre. Fig. 2 shows the specifications of the resources and Fig. 3 shows the requirement of each resource by activities. The total resource requirement is Rs.27,320 for the Total Project out of which Oil-Grade-1A requires Rs.19000 (i.e. 190 litres).

The main decision to be made by the Project Manager is :

"How many litres of Oil-Grade-1A to be ordered each day"?

3. Wagner-Whitin Model for Inventory Control :

Inventory Models are the most researched models in Operations Research [2,3,4]. Wagner-Whitin (W-W) Model is a multi period deterministic inventory model for a single item [1]. This section describes briefly W-W Model.

Important assumptions of W-W Model are :

* Demand is deterministic
* Demand varies from period to period
* Inventory level is revised periodically.
* Stock is replenished instantaneously at the beginning of the period
  (Even though lag may be considered in the modified model)
* No shortage is allowed
* Finite number of periods.

power-unit Plan 19-09-92 RESOURCE VIEW	Capacity	Unit Cost	Accrual Period
- mechanics	10*8h/d	R5.00/h	During
- technicians	5*8h/d	R10.00/h	During
- Oil-Grade-1A	Budget	na	During

F2/HELP F10/MENU

FIG 2 : RESOURCE SPECS

power-unit Plan 19-09-92 SPREADSHEET VIEW	Total	mechanics	technicia	Oil-Grade
* + power-unit Plan {19ed}	R27320	R4800	R3520	R19000
- 1:disassemble power un[3ed]	R2960	8*8h/d		R2000
- 2:test & repair instru[9ed]	R4440		2*8h/d	R3000
- 3:clean main frame [2ed]	R1160	2*8h/d		R1000
- 4:procure new assembly[1ed]	R40	1*8h/d		
- 5:pull old assembly [2ed]	R160	2*8h/d		
- 6:clean, inspect & rep[4ed]	R5600	8*8h/d	1*8h/d	R4000
- 7:calibrate instrument[8ed]	R3280		2*8h/d	R2000
- 8:inspect & repair [1ed]	R3080	2*8h/d		R3000
- 9:change tangs [1ed]	R40	1*8h/d		
- 10:INSTALL NEW ASSEMBL[2ed]	R1080	1*8h/d		R1000
- 11:assemble & test [5ed]	R3600	8*8h/d		R2000
- 12:check fittings [3ed]	R240	2*8h/d		
- 13:retrofit check [1ed]	R120	1*8h/d	1*8h/d	
- 14:safety inspection [1ed]	R120	1*8h/d	1*8h/d	
- 15:system check [2ed]	R1400	1*8h/d	2*8h/d	R1000

Current Plan:R3000
2:test & repair instruments [9ed]
Planned Start:01-09-92 End:09-09-92 Slack:0d Total:R4440
F2/HELP F10/MENU

FIG 3 : RESOURCES BY ACTIVITIES

Define for period i = 1,2,..... N.

z_i = amount ordered

d_i = amount demanded

x_i = entering inventory at the beginning of period i

h_i = holding cost of one unit of item carried forward from i to i+1 period.

K_i = set up cost

$c_i(z_i)$ = marginal purchasing cost function for given z_i

$c_i(z) = S_i K_i + c_i(z_i)$

where $S_i = \begin{cases} 0 & z_i = 0 \\ 1 & z_i > 0 \end{cases}$

Typical Holding cost for period i may be proportional to

$x_{i+1} = x_i + z_i - d_i$

Holding cost for period i = $h_i x_{i+1}$

(Other types of holding cost function may also be considered).

The W-W Model is schematically depicted in Fig 4.

The model is formulated as Dynamic Programming Model [5,6].

Let $f_i(x_i)$ = minimum inventory cost for periods for i, i+1, ..., N.

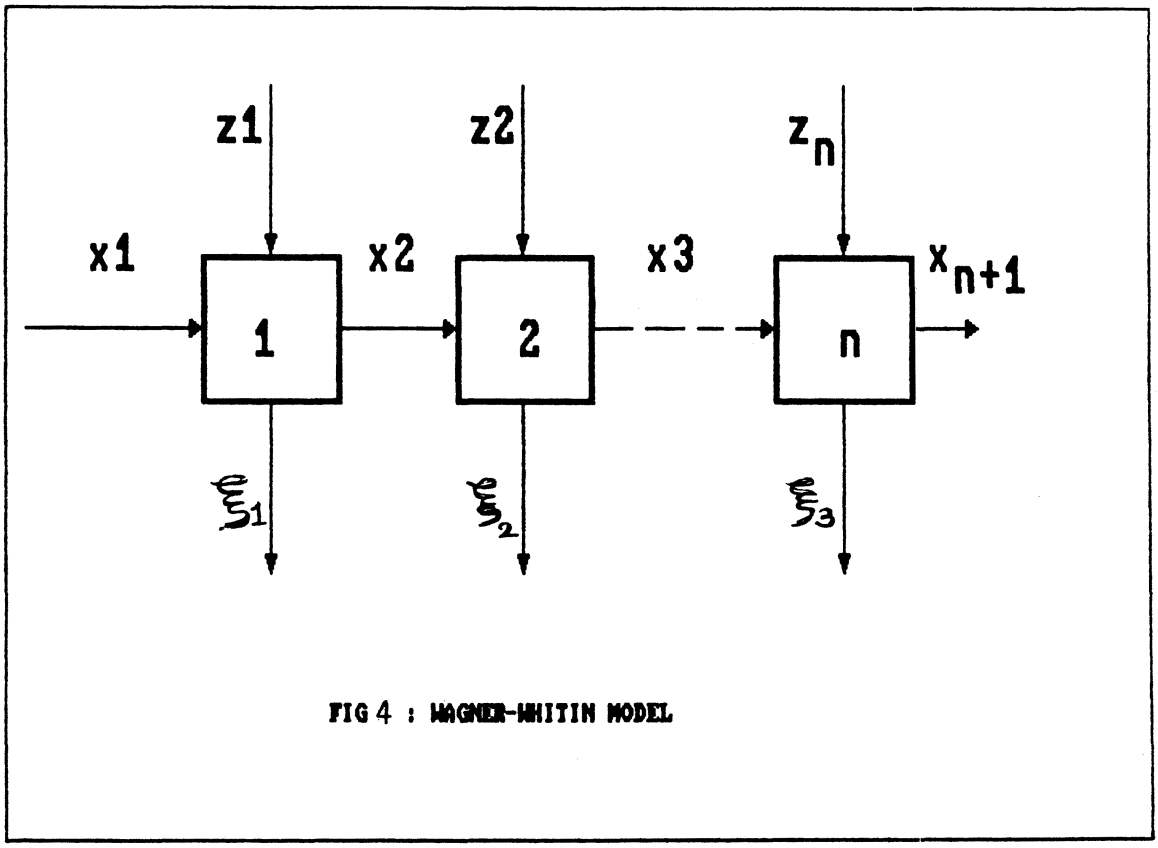


FIG 4 : WAGNER-WHITIN MODEL

Hence, using the backward recursion equation :

$$\left.\begin{aligned} f_N(x_N) &= \min\{c_N(z_N)\} \\ & z_N + x_N = d_N \\ & z_N \geq 0 \\ f_i(x_i) &= \min\{c_i(z_i)+h_i(x_i+z_i-d_i)+f_{i+1}(x_i+z_i-d_i)\} \\ & d_i \leq x_i + z_i \leq d_i + \ldots + d_N \\ & z_i \geq 0 \end{aligned}\right\} \quad (1)$$

Similarly, using the forward recursion equation :

Let $f_i(x_{i+1})$ = min inventory cost for periods 1,2 ... i

where x_{i+1} = Inventory at the end of period i

Therefore forward recursion equation is,

$$\left.\begin{aligned} f_i(x_2) &= \min\{c_1(z_1) + h_1 x_2\} \\ & 0 \leq z_1 \leq d_1 + x_2 \\ f_i(x_{i+1}) &= \min\{c_i(z_i)+h_i x_{i+1}+f_{i-1}(x_{i+1}+d_i-z_i)\} \\ & i = 2,3,\ldots.N \end{aligned}\right\} (2)$$

The backward recursion model (1) and forward recursion model (2) are equivalent. For computation any of the models can be used.

4. Resource Profile and W-W Model

The assumptions stated in Section 3 for W-W Model are valid for the multiperiod resource profile of an item in the Project Management techniques. Hence it is possible to integrate Project Management and W-W Model. The output of the resource analysis of a specific total resource can be obtained by the Resource Analysis. This output will form the

input to W-W Model to get the optimal inventory policy for the resource. The composite model is schematically shown in Fig 5.

4.1 Resource Profile of Maintenance of Electrical Equipment and Optimal Policy of Oil-Grade-1A :

Fig. 6 shows the Resource Profile of Oil-Grade-1A for 19 days. From the demand figures, the Project Manager may find it difficult to decide on how much to buy and when to buy.

The Wagner-Whitin Model is solved using the data shown in Fig.6 and the parameter values for W-W Model as :

K_i = ordering cost = Rs.20/order

c_i = purchasing cost = Rs.100/litre of Oil-Grade-1A

h_i = holding cost = Rs.1/litre/day

It is assumed that there is no restriction on the quantity of oil that can be purchased and also there is no restriction on the storage capacity of the oil at the project-site. (Even though such restriction can also be included in W-W Model).

The solution of W-W Model is also shown in Fig.6. It may be seen that there are only 8 orders for the total Oil-Grade-1A requirement of 190 litres. This optimal policy is based on the minimum total cost for solution using Dynamic Programming formulation as described in Section 3, considering ordering cost, purchase cost and holding cost.

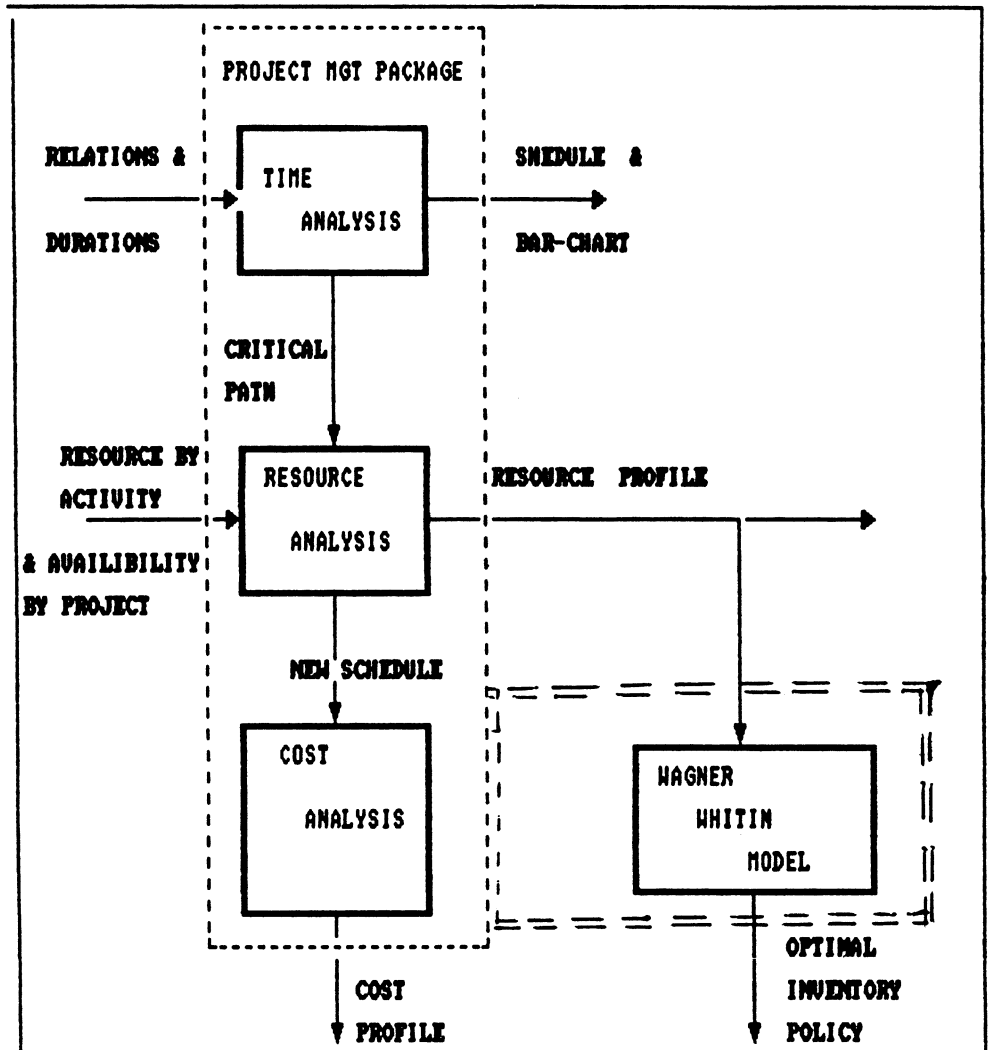
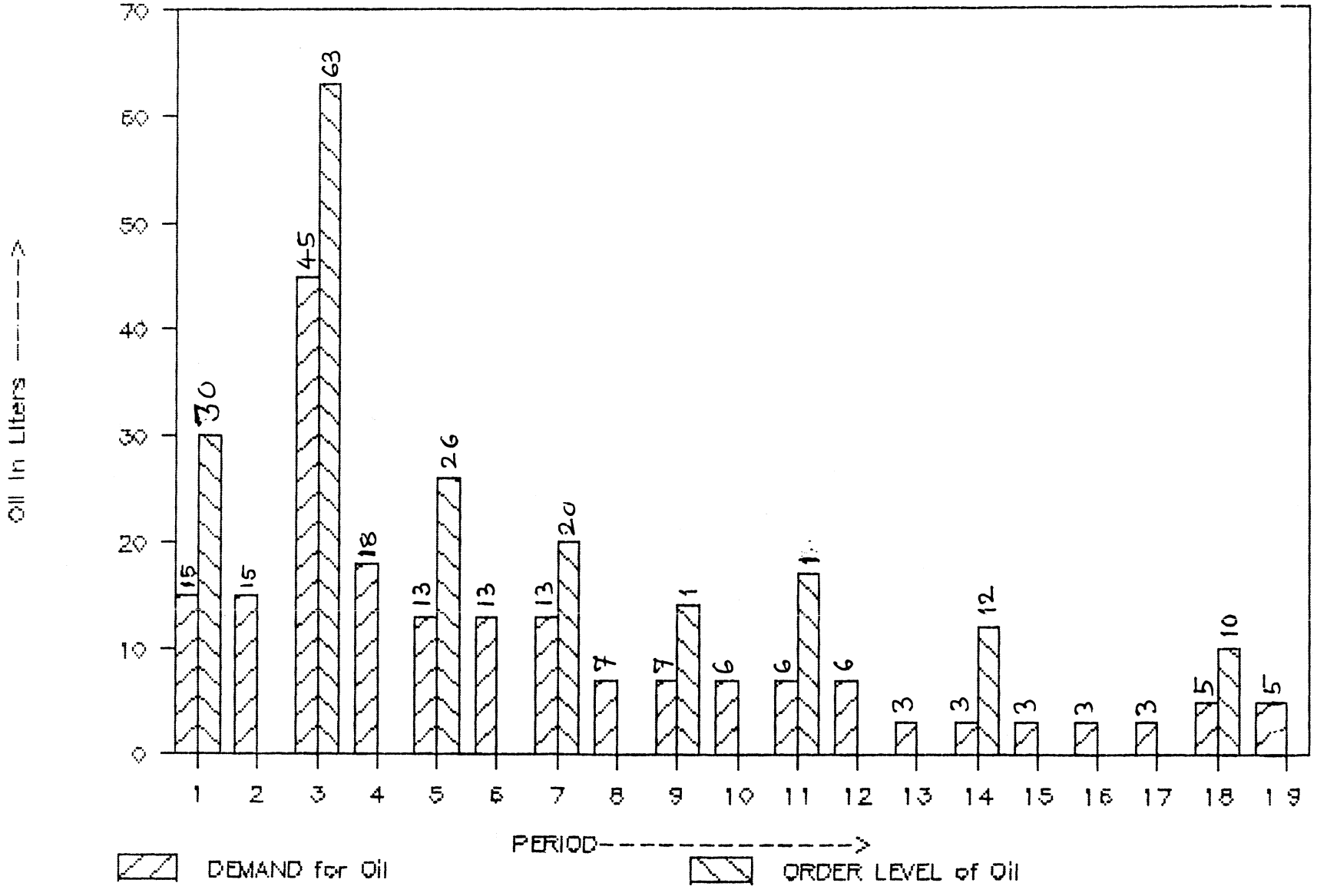


FIG 5: COMPOSITE MODEL OF PROJ MGT & W-W MODEL

FIG.6 : DEMAND & ORDER_LEVEL of Oil—Grade—1A



5. Concluding Remarks

The Project Management package used in this work is InstaPlan III from Wipro Systems Ltd. [7]. The Wagner-Whitin Model was solved by an Operations Research Software Package "Quant Systems" (QS) ver 1.0 supplied by Prentice-Hall Inc., USA [8].

Following extensions/modifications are possible to this composite model :

* Other deterministic inventory models may be tried instead of Wagner-Whitin Model.
* Stochastic Inventory Models can also be considered, if the problem requires such models.
* Manpower planning for "RATE-RESOURCE" may be analysed.
* Interfacing the Project Management packages (like InstaPlan III) and Dynamic Programming Packages (like QS) may be attempted so that it is a complete package to the Project Manager.
* Multi-item Resource Analysis and Inventory Policy are complex. They can be considered for further research.

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