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FORMULATIONS FOR THE MULTI-ITEM LOT SIZING PROBLEM WITH JOINT REPLENISHMENTS - SOME EXTENSIONS

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Abstract

A modified formulation of the multi-item lot sizing problem with joint replenishments is discussed. The new formulation results in the reduction in the number of constraints and consequently possible reduction in the computational effort.The formulation is extended to the case when there is an upperbound on the production for a given set up.

1. INTRODUCTION

The multi-item lot sizing problems with joint replenishments have been studied by a few authors in the past. Dynamic Programming algorithms were used to solve these problems (Zangwill[12], Veinott [11], Kao [6]). The other approaches include implicit enumeration (Erenguc [3]), dual ascent with branch and bound (Robinson and Gao [10]) and strong cutting plane algorithm with branch and cut (Raghavan and Rao [8]). A heuristic method of solution for certain cost structures were also given (Joneja [5]). More recently. Raghavan and Rao [9] presented different formulations of the multi-item lot sizing problems with joint replenishments and a comparitive evaluation of their usefulness from a computational view point was made.

In this paper an attempt is made first to reduce the number of constraints in the standard formulation and then extended to the situations where there is an upperbound on the production for a given set-up. There are plans to present computational results in due course.

REVIEW

The multi-item lot sizing problem with joint replenishments, as stated by Raghavan and Rao [9] is 'as follows:

Given the demand forecast for the K products over a finite horizon of T time periods,the objective is to find a lot schedule that meets the demand of each item in every period and minimizes the total cost over the entire horizon. A joint set up cost, j_t, is incurred in a period t, if any one of the items is produced in that period. An individual set-up cost, c_{tk}, is incurred in period t if there is any production of product k in period t. A marginal cost of production for item k in period t, p_{tk}, and a marginal cost of holding inventory of item k at the end of period t, h_{tk}, are also incurred. The variables are as defined in [9].

The standard formulation of the problem is:

Minimize

$$\sum_{t=2}^{T} j_t z_t + \sum_{t=1}^{T}\sum_{k=1}^{K} p_{tk} y_{tk} + \sum_{t=2}^{T}\sum_{k=1}^{K} c_{tk} x_{tk} + \sum_{t=1}^{T-1}\sum_{k=1}^{K} h_{tk} s_{tk}$$

subject to

(a)	$y_{1k} - s_{1k} = d_1^k$	$k = 1,\ldots, K$
(b)	$y_{tk} + s_{t-1,k} - s_{tk} = d_t^k$	$t = 2,\ldots,T-1$
(c)	$y_{Tk} + s_{T-1,k} = d_T^k$	$k = 1,\ldots,K$
(d)	$y_{tk} \le d_{tT}^k x_{tk}$	$t = 2,\ldots,T$; $k = 1,\ldots,K$
(e)	$x_{tk} \le z_t$	$t = 2,\ldots,T$; $k = 1,\ldots,K$
(f)	$y_{tk} \ge 0$	$t = 1,\ldots,T$; $k = 1,\ldots,K$
(g)	$s_{tk} \ge 0$	$t = 1,\ldots,T-1$; $k = 1,\ldots,K$

(h) $x_{tk} \in \{0,1\}$ $t = 2,\ldots,T$

$k = 1,\ldots,K$

(i) $z_t \in \{0,1\}$ $t = 2,\ldots,T$

The total number of constraints in the problem , as given by (a),(b), (c), (d) and (e) is:

$$K + K(T-1) + K + K\,T + K\,T = K\,(3T + 1)$$

This is reformulated as multi commodity problem considering the quantity of product k produced in period i to meet the demand in period t, as a variable. The marginal cost of producing one unit of product k in period i and carrying the unit to period t was also considered.

Rabinson and Gao [10] adopted the facility location formulation. The JRP was also formulated as a shortest path problem on an acyclic network by Joneja [4]. Raghavan and Rao [9] presented computation experience with a working formulation which has a block diagonal structure.

MODIFIED FORMULATION

The standard formulation of the JRP is modified to reduce the number of constraints from K(3T+1) to K (2T+1) + T, i.e. a reduction of T (K-1).

This is achieved by replacing the following constraints:

$$x_{tk} \le Z_t \quad ; \quad t = 1,2,\ldots,T \quad k = 1,2,\ldots,K \qquad (1)$$

by

$$x_{t1} + x_{t2} + \ldots x_{tK} \le K.\ Z_t \quad ; \quad t = 1,2,\ldots,T \quad .. \ (2)$$

It can easily be seen that constraints (1) and (2) are equivalent. The reduction in the number of constraints could be substantial if the number of time periods,T,or the number of items,K,or both, are large.

The constraint set (1) appears in the multicommodity formulation, in the facility location formulation and in the shortest path formulation.

Solution for some sample problems are now being worked out with this modified formulation.

EXTENSIONS

In multi item lot sizing problem with joint replenishment, situations frequently arise where there is an upper bound on the number of units that could be produced per set up. If the number of units produced is more than this upperbound, then it warrants a second set up.In machine shop situation this may involve replacing the tool. In cigerrette manufacturing situation,it may involve changing the cutting knife.

Here it is assumed that for product k, the maximum number of units that could be produced in a time period from the first setup is $U_k^{(1)}$. If the number of units to be produced is more than $U_k^{(1)}$, then a second setup is required and the maximum number that could be produced from the second setup is $U_k^{(2)}$ .If the number of units to be produced in the time period is more than $U_k^{(1)} + U_k^{(2)}$, then a third setup is required. Let $U_k^{(n)}$ denote the maximum number of units that could be produced for n^{th} setup.Let the maximum number of setups to be considered for product k in time period t be N_k^t, which is determined as:

$$U_k^{(1)} + U_k^{(2)} + \ldots + U_k^{(N_k^t - 1)} < d_k^t + d_k^{t+1} + \ldots d_k^T \leq$$

$$U_k^{(1)} + U_k^{(2)} + \ldots\ldots U_k^{(N_k^t)}$$

It is also assumed that the set ups for a product in time period t are independent of the setups in other periods.

Let us define ,as in the standard formulation of multi item lot sizing problem with joint replenishments, the following parameters and variables.

For $t = 1,2,3, \ldots, T$

$k = 1, 2, 3, \ldots, K$

d_t^k = forecasted demand for product k in time period t

d_{tu}^k = sum of demands of product k from period t through period u inclusive, t = t+1,t+2,....,T

j_t = joint setup cost in time period t if any one of the products is produced in time period t

c_{tk} = individual setup cost in time period t if product k is produced in time period t

p_{tk} = marginal cost of production for product k in time period t

h_{tk} = marginal cost of holding one unit of product k at the end of period t

$U_k^{(n)}$ = maximum number of units of product k that could be produced in time period t for the nth set up

$n = 1,2,3,\ldots, N_k^t$

Z_t = 0 if there is no joint set up cost incurred in time period t

= 1 if there is a joint set up cost incurred in time period t.

$X_{tk}^{(r)}$ = 1 if there are r setups for product k in time period t

= 0 if there are less than r setups for product k in time period t

Y_{tk} = number of units of product k produced in time period t

s_{tk} = number of units of inventory of product k carried at the end of period t,

$t = 1,2,3,\ldots (T-1)$.

The formulation of the problem is given below:

Minimize

$$\sum_{t=1}^{T} j_t z_t + \sum_{t=1}^{T}\sum_{k=1}^{K} p_{tk} Y_{tk} + \sum_{t=1}^{T}\sum_{k=1}^{K} C_{tk} \left(\sum_{r=1}^{N_k^t} X_{tk}^{(r)}\right) + \sum_{t=1}^{T-1}\sum_{k=1}^{K} h_{tk} s_{tk}$$

subject to:

a) $Y_{1k} - s_{1k} = d_{1k}$; $k = 1,2,\ldots K$

b) $Y_{tk} + s_{t-1,k} - s_{t,k} = d_t^k$; $t = 2,3,\ldots T-1$; $k = 1,2,\ldots K$

c) $Y_{TK} + s_{T-1,k} = d_T^k$; $k = 1,2,\ldots K$

d) $Y_{tk} \le U_k^{(1)} X_{tk}^{(1)} + U_k^{(2)} X_{tk}^{(2)} + \ldots + U_k^{N_k^t} . X_{tk}^{N_k^t}$

e) $X_{t1}^{(1)} + X_{t2}^{(1)} + \ldots X_{tk}^{(1)} \le K\ Z_t$; $t = 1,2,\ldots T$

f) $X_{tk}^{(n+1)} \le X_{tk}^{(n)}$; $n = 1,2,\ldots(N_k^t - 1)$; $t = 1,2,\ldots T$; $k = 1,2,\ldots K$

g) $Y_{tk} \ge 0$; $t = 1,2,\ldots T$; $k = 1,2,\ldots K$

h) $s_{tk} \ge 0$; $t = 1,2,\ldots (T-1)$; $k = 1,2,\ldots K$

i) $Z_t \in \{0,1\}$; $t = 1,2 \ldots T$

j) $X_{tk}^{(n)} \in \{0,1\}$; $t = 1,2 \ldots T$; $k = 1,2,\ldots K$; $n = 1,2,\ldots N_k^t$

Constraint sets (a),(b), and (c) require that the demand of every product be met in every time period and that there be no final inventory. Constraint set (d) relates production of product k in time period t with the number of setups in that period. Constraint set (e) ensures that there is no individual setup in a period in which there is no joint replenishment. Constraint set (f) ensures that in any period, for product k there will not be (n+1)th set up unless there is a n th setup.Constraint sets (g),(h),(i) and (j) are self explanatory.

This can also be formulated as multicommodity flow problem,facility location problem and as a shortest path problem.

Some sample problems will be selected and the computational experiences will be reported later.

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