MEDIA PLANNING - MAXIMIZING GROSS OTS

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Abstract

The problem of generating a cinema schedule for advertisement to maximize gross OTS (Opportunities-To-See) attracted the attention of media plan researchers. The development of a cinema schedule so that desired Reach is achieved in each town and the total cost of advertisement does not exceed a given budget was formulated as a mathematical programming problem. The problem has a linear objective function with nonlinear constraints and integer variables. A heuristic procedure was developed by researchers in the past to solve the problem. In this paper a nested dynamic programming formulation is given to obtain an optimal solution to the problem. The structure of the optimal solution and solution procedure for a special case is also discussed.

1. INTRODUCTION

As per Kotler [3], the major decision problems that a product advertiser encounters are :

a. determination of the media classes and the media vehicles in a media class that could possibly be used for conveying the message, and

b. determination of frequency with which each of the potential vehicles are to be used during the advertising campaign

There are various measures of effectiveness of an advertisement, like reach, gross opportunities-to-see (GOTS), average cost per person reached etc.. In the past, researchers developed models and given solution procedures to optimize the effectiveness of an advertisement (see foe instance, [1}, [2]) . Sarla Achuthan [4], developed a model for cinema plan that maximizes gross OTS for an entire region subject to a budget constraint. The objective was to develop a cinema plan that specifies the theatres where the advertisements are to be screened and the number of weeks for which the advertisements are to be screened in the chosen theatres. Sarla Achuthan [4] developed a mathematical programming model to determine the number of screening weeks in each theatre of the towns in a region, which will maximize the gross OTS. The constraints considered are, the total budget, achieving a specified minimum reach in each town, and ensuring that in every town the proportion of people who see the advertisement for at least a pre-specified number of times is a proportion of the maximum possible reach in that town. There are additional lower and upper bound constraints on the number of screening weeks in a theatre, if that theatre is chosen for screening. This problem is shown to be equivalent to a integer LP problem with a reduced set of constraints. This is based on the observation that there is a lower bound on the total number of screenings in a town below which the problem has no feasible solution and this lower bound could be easily identified. The equivalent problem has the same objective function as the first problem. The constraints consist of total availability of budget, lower limits on the total number of screenings in each town, and lower and upper bounds on the number of screenings in the theatres chosen for screening in a town. As the number of variables involved is large, the author suggested a heuristic iterative procedure to arrive at a solution to the problem. Empirical results were also presented. In this paper, a nested dynamic programming solution is given. Solution procedures for some special cases are also discussed.

2. FORMULATION

The notation and mathematical formulations as given by Sarla Achuthan [4] are given below:

Notation:

Let

N denote the number of towns in the region ;

M_j denote the number of potential theatres in j^{th} town $j = 1,2,\ldots,N$;

R_j^* denote the desired reach in j^{th} town $j = 1,2,\ldots,N$;

C_{kj} denote the cost of screening per week in the k^{th} theatre of j^{th} town; $j = 1,2,\ldots,N$; $k = 1,2,\ldots,M_j$;

i denote the random variable representing the number of visits to a theatre by a person in the target audience, $i = 0,1,\ldots,n_j$;

n_j denote the maximum value that the random variable i takes in the j^{th} town ;

f_{ij} denote the probability that a person in the target audience makes 'i' visits to the theatres in the j^{th} town;

U_{kj} denote the upper bound on the screening weeks for the k^{th} theatre in the j^{th} town ;

D_j denote the number in the target audience of the j^{th} town;

L_j denote the lower bound of the screening weeks in a theatre if that theatre is chosen. That is, if a theatre is chosen, the advertisement should be screened for at leat L_j weeks in that theatre of the j^{th} town ;

C denote the total budget for the region.

DECISION VARIABLES:

W_{kj} denote the number of screening weeks in the k^{th} theatre of the j^{th} town, $k = 1,2,\ldots,M_j$ and $j = 1,2,\ldots,N$.

W_j denote the total number of screening weeks in the j^{th} town,

$$W_j = \sum_{k=1}^{M_j} W_{kj}$$

The mathematical formulation is

Problem P1:

$$\text{Maximize} \quad \sum_{j=1}^{N} \sum_{i=1}^{n_j} \frac{i\ f_{ij}}{52M_j} W_j D_j \qquad \ldots\ldots\ldots\ldots \qquad (1)$$

subject to

$$\sum_{j=1}^{j=N} \sum_{k=1}^{M_j} C_{kj} W_{kj} \leq C \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

$$R_j = [1-f_{0j}] - \sum_{i=1}^{n_j} \frac{f_{ij} \binom{52M_j - W_j}{i}}{\binom{52M_j}{i}} \geq R_j^*$$

$$j=1,2,\ldots,N \quad \ldots\ldots\ldots \quad (3)$$

$$G_{k_o}(W_j) = \sum_{k=k_o}^{n_j} \sum_{i=k}^{n_j} \frac{f_{ij}}{\binom{52M_j}{i}} \binom{52M_j - W_j}{i-k} \binom{W_j}{k} \geq [1-f_{0j}]\, p_j$$

$$j = 1,2,\ldots,N \quad \ldots.(4)$$

where k_o is a pre-specified integer and p_j's are pre-specified proportions.

$W_{kj} \leq U_{kj}$ for all j,k .... (5)

$W_{kj} = 0$ or $W_{kj} \geq L_j$ .... (6)

W_{kj} 's are all integers.

The objective function (1) is the expression for gross OTS which needs to be maximized.

Inequality (2) represents the budget constraint. Inequality (3) ensures that the desired reach is achieved in every town of the region. Constraint (4) ensures that in every town the proportion of the people who see the advertisement at least k_o times is a proportion of maximum reach of that town.

Constraint (5) ensures an upper bound on the screening weeks.

Constraint (6) ensures that if a theatre is chosen, the advertiisement should be screened for atleast some minimum number of weeks.

It may be noted that the gross OTS depends only on the total number of screenings in a town and not on the number of screenings in a particular theatre.

It is also noted that Reach R_j is a strictly monotonic increasing function of the total number of screening weeks, W_j in town j. Also, the cost function is linear, additive, separable and a monotonic increasing function of W_j . Using the above, some special features of problem P1 are exploited.

The solution set for reach (equation 3) and OTS (equation 4) constraints can be determined by total screening weeks in that town i.e., W_j's. The lower bounds W_j° for W_j can be determined by the method of interval bisection.

We start with $W_j = [\ 52\ M_j\ /2]$. If the reach calculated for W_j is less than R_j^*, then increase W_j to $[\ 52\ M_j/2] + [52\ M_j/4]$. Depending on the values of reach, use W_j values in the left interval or right interval. By this method we get the lower bound on W_j which satisfies minimum reach and OTS constraints.

Noting that $W_j = \sum_{k=1}^{n_j} W_{kj}$, the solution to constraints (3) and (4) will be of the form W_{kj} such that $\sum_{k=1}^{M_j} W_{kj} \geq W_j^\circ$.

Then, the equivalent form of (P1) is

Problem P2:

Maximize $$\sum_{j=1}^{N} \sum_{i=1}^{n_j} \frac{i f_{ij} W_j D_j}{52 M_j}$$

subject to:

$$\sum_{j=1}^{j=N} \quad \sum_{k=1}^{M_j} C_{kj} W_{kj} \leq C$$

$$\sum_{k=1}^{M_j} W_{kj} \geq W_j^{\circ} \qquad \text{for } j = 1,2,\ldots, N$$

$$W_{kj} \leq U_{kj}$$

$$W_{kj} \geq L_j \quad \text{or} \quad W_{kj} = 0$$

An approximate solution method has been given by Sarla Achuthan [4] when $L_j = L$ for all j. The method consists of an iterative procedure. The value of W_j's are obtained by using the average cost of screening in a town. After getting the values of W_j's, the values for W_{kj}, the number of screenings in the individual theatres of a town are determined using the following formulation:

Problem P3:

$$\text{Minimize} \quad \sum_{j=1}^{N} \sum_{k=1}^{M_j} C_{kj} W_{kj}$$

subject to:

$$\sum_{k=1}^{M_j} W_{kj} \geq W_j^{(t)} \quad , \; j = 1,2,\ldots.,N$$

$$W_{kj} \leq U_{kj}$$

$$W_{kj} = 0 \text{ or } W_{kj} \geq L$$

W_{kj}'s are integers; $k = 1,2,\ldots, M_j$, $j = 1,2,\ldots,N$

$W_j^{(t)}$ are the lower bounds on the total number of screenings in j th town obtained at the t th iteration while solving problem P2.

Problem (P3) is decomposed into N bounded knapsack problems. The solutions for W_{kj} are then used to arrive at the average cost in the next iteration. The procedure stops when either the minimum cost exceeds the budget given for the region or the screening policy in two consecutive iterations remains the same. The author also pointed out that this solution procedure does not guarantee an optimal solution.

3. NESTED DYNAMIC PROGRAMMING APPROACH

As noted earlier, gross OTS depends only on the total number of screenings in a town and not on the number of screenings in individual theatres.

Problem P2 can now be considered as N sub-problems, one for each town. Suppose B_j is the amount available for allocation to town j. Then, for town j, the sub problem is :

Problem P4:

$$\text{Maximize} \sum_{i=1}^{n_j} \frac{i\ f_{ij}(\sum_{i=1}^{M_j} W_{kj})}{52M_j} D_j$$

subject to:

$$\sum_{k=1}^{M_j} C_{kj}\ W_{kj} \le B_j$$

$$\sum_{j=1} W_{kj} \ge W_j^0$$

$$W_{kj} \le U_{kj}$$

$$W_{kj} \ge L_j \quad \text{or} \quad W_{kj} = 0\ , k = 1,2,\ldots,M_j$$

Let $$G_j = \sum_{i=1}^{n_j} \frac{i\ f_{ij}}{52M_j} D_j$$

= GOTS per screening in any theatre in town j.

Then the objective function of the sub problem is

$$\text{Maximize } G_j \sum_{k=1}^{M_j} W_{kj} = G_j \ \text{Max} \sum_{k=1}^{M_j} W_{kj}$$

Let $f_s(b)$ = Maximum number of screenings that are possible using theatres 1 to s and using budget b; $1 \leq s \leq M_j$ and $b = 0,1,2,\ldots,C$.

The recursive equation is:

$$f_{s+1}(b) = \text{Max} \quad \{ Y_{s+1,j} + f_s(b - C_{s+1,j}\, Y_{s+1,j}) \}$$

$$Y_{s+1,j} = 0 \text{ or}$$
$$L_{s+1,j} \leq Y_{s+1,j} \leq U_{s+1,j} \text{ and}$$
$$C_{s+1,j}\, Y_{s+1,j} \leq b$$

with the initial condition $f_0(b) = 0$ for $b = 0,1,2,\ldots,C$.

If $f_{M_j}(b) < W_j^0$, then problem P2 is infeasible and f_{M_j} is set to $-\infty$.

If $f_{M_j}(b) \geq W_j^0$, then the maximum value of the subproblem is $G_j\, f_{M_j}(b)$.

Now, we solve the master problem for the region.

Suppose a is the amount available for allocation to towns 1 to j. The recursive equation is:

$$F_j(a) = \underset{0 \leq Y \leq a}{Max} \quad [f_{M_j}(Y) + F_{j-1}(a-Y)]$$

$a = 0,1,2,\ldots,C$ and $j = 1,2,\ldots,N$ with the initial condition

$F_j(0) = 0$ for $j = 1,2,\ldots,N$ and

$F_0(Y) = 0$ for all Y.

The optimal objective function value for Problem P1 is given by $F_N(C)$.

Solution procedure: For any given budget C, the optimal soltion values W_{kj} could be obtained by back tracking as is done in any dynamic programming solution method.

4. PROPERTIES OF SUB PROBLEM SOLUTIONS

We now consider a special case where the optimal solution can be found without using Dynamic Programming to solve the sub problems. The master programme gives, for a total budget for the region, the optimal budget allocation for each town. For a given budget allocation for a town, we need to find the maximum number of screenings possible in the theatres of the town. If we can get for each town the relationship of the total number of screenings in the town and the corresponding minimum cost, then this relation could be used to find, for a given budget allocation to a town, the optimum number of screenings and their allocation to the theatres.

We consider some special properties of the solutions to the sub problem.

For any town, the general formulation of the sub problem is given below omitting the symbol j for town:

Let Y_i = number of screenings in theatre i; i = 1,2,....,M.

Problem P5:

$$\text{Minimize} \quad \sum_{i=1}^{M} C_i \, Y_i$$

subject to:

$$\sum_{i=1}^{M} Y_i = W$$

$$Y_i \leq U_i \;;\; i = 1,2,\ldots,M$$

$Y_i \geq L_i$ or 0 ; $i = 1,2,\ldots,M$

Y_i's are integers

where

M : number of theatres in the town

U_i : upper limit on the number of screenings in theatre i

L_i : lower bound on the number of screenings in theatre i, if chosen for screening

W : total number of screenings in all the theatres of the town

We assume that

$L = L_i$ for all $i = 1,2.\ldots,M$

and the theatres are numbered in increasing order of cost

i.e., $C_1 < C_2 < \ldots\ldots < C_M$.

We also assume that $U_i > L$ for all i.

Let { Y_t^* } be an optimal solution to Problem P5.

We prove the following properties of the optimal solution to Problem P5.

Lemma 1: For atmost one i, the value of Y_i^* will be strictly between the lower and upper bounds i.e., $L < Y_i^* < U_i$ for atmost one i.

Proof: Suppose for some i and j such that $i < j$, $L < Y_i^* < U_i$ and $L < Y_j^* < U_j$.

Consider a new solution

$$Y_k = Y_k^* \qquad \text{for } k \neq i \text{ or } j$$

$$Y_i = Y_i^* + 1$$

$$Y_j = Y_j^* - 1$$

The new solution is feasible and the corresponding value of the objective function will reduce from the optimum cost by

$(C_j - C_i) > 0$ which is not possible since $\{ Y_t^* \}$ is optimal.
Hence, $L < Y_i^* < U_i$ for atmost one i.

Lemma 2: If $Y_i^* = L$ for any i, then $Y_j^* = 0$ or L for all $j > i$.

Proof: Suppose $Y_i^* = L$ and for some $j > i$, $Y_j^* > L$.

Consider a new solution,

$$Y_k = Y_k^* \quad \text{for } k \neq i \text{ or } j$$

$$Y_i = L + 1$$

$$Y_j = Y_j^* - 1$$

The new solution is feasible and the corresponding value of the objective function will reduce from the optimum cost by $(C_j - C_i) > 0$ which is not possible since $\{ Y_t^* \}$ is optimal.
Hence, $Y_j^* = 0$ or L for $j > i$.

Lamma 3: If $Y_i^* = 0$, then for any $j > i$, $Y_j^* = 0$ or greater than U_i .

Proof: Suppose $Y_i^* = 0$ and for some $j > i$, $L \leq Y_j^* \leq U_i$.

Consider the new solution

$$Y_k = Y_k^* \text{ for } k \neq i \text{ or } j$$

$$Y_i = Y_j^*$$

$$Y_j = 0$$

The new solution is feasible and the corresponding value of the objective function will reduce by $(C_j - C_i) Y_j^*$, which is not possible since $\{ Y_t^* \}$ is optimal.
Hence $Y_j^* = 0$ or greater than U_i .

Remark 1: Lemmas 2 and 3 imply that if $Y_i^* = L$ and $Y_j^* = L$, then for all k such that $i < k < j$, $Y_k^* = L$.

From Lemmas 1,2 and 3, we get the structure of the optimal solution $\{ Y_t^* \}$ as:

$$Y_j^* = 0 \text{ or } U_j \quad \text{for } 1 \le j \le k \quad \ldots \quad (8)$$

$$Y^*_{k+1} \text{ will satisfy } L \le Y^*_{k+1} \le U_{k+1} \quad \ldots \quad (9)$$

$$Y_j^* = L \quad \text{for } k+2 \le j \le t \quad \ldots \quad (10)$$

$$Y_j^* = 0 \quad \text{for } t+1 \le j \le M \quad \ldots \quad (11)$$

It is possible that one or more of the sets given in (8), (9) or (10) could be empty. If $Y_j^* = 0$ for any j in (8), then by Lemma 3, set given in (10) will be empty.

We now consider a special case where the theatres are numbered as per increasing order of cost, and the upper bounds are non increasing i.e., $C_1 < C_2 < \ldots\ldots < C_M$ and $U_1 \ge U_2 \ge \ldots\ldots \ge U_M$.

For this special case, we prove the following Lemma, which gives the structure of the optimal solution $\{ Y_t^* \}$, $t = 1,2,\ldots\ldots,M$.

Lemma 4 : If $Y_i^* = 0$ then $Y_j^* = 0$ for all $j > i$.

Proof: Suppose $Y_j^* \ge L$ for some $j > i$.

Consider the new solution,

$$Y_k = Y_k^* \quad \text{for } k \neq i \text{ or } j$$

$$Y_i = Y_j^*$$

$$Y_j = 0$$

As $U_i \ge U_j$ and $Y_j^* \le U_j$, the solution is feasible. Since $C_j > C_i$, the optimum value of the objective function will reduce by $(C_j - C_i) Y_j^* > 0$, which is not possible .

Hence $Y_j^* = 0$.

Remark 2: From Lemma 4, we get the structure of the optimal solution for this special case as

$$Y_j^* = U_j \ , \quad j = 1,2,\ldots,k \qquad \ldots \quad (12)$$

$$Y^*_{k+1} \text{ will satisfy } L \le Y^*_{k+1} \le U_{k+1} \qquad \ldots \quad (13)$$

$$Y_j^* = L \qquad \text{for } k+2 \le j \le t \qquad \ldots \quad (14)$$

$$Y_j^* = 0 \qquad \text{for } t+1 \le j \le M \qquad \ldots \quad (15)$$

As in Remark 1, it is possible that one or more of the sets (12), (13) and (14) could be empty.

We now state a procedure for finding the optimal solution and show that this procedure gives the optimal solution.

Procedure for obtaining the optimal solution:

Obviously, if $W < L$, the problem is infeasible. So assume that $W \ge L$.

Let $k \ge 0$ be the largest integer, and t be the smallest integer greater than or equal to $k+2$ for which

$$\sum_{i=1}^{k} U_i + Y^*_{k+1} + \sum_{i=k+2}^{t} L = W \qquad \ldots\ldots \quad (16)$$

and $L \le Y^*_{k+1} \le U^*_{k+1}$

where $W \ge L$, is the total number of screenings in all theatres of the selected town. The expression $\sum_{i=k+2}^{t} L$ may not exist.

The process of finding k and t involves a simple search procedure. One such procedure is detailed below.

Step 1: Find j such that

$$W = \sum_{i=1}^{j} U_i + R \qquad \text{where } 0 \le R < U_{j+1}$$

If $R = 0$, set $k = j - 1$, $Y^*_{k+1} = U_{k+1}$ and t does not exist. Stop.

If $R \geq L$, set $k = j$, $Y^*_{k+1} = R$ and t does not exist. Stop.

If $R < L$, go to step 2.

Step 2: Set $s = 0$.

Step 3: Set $s = s+1$. Find j such that

$$W - s\,L = \sum_{i=1}^{j} U_i + R \quad \text{where } 0 \leq R < U_{j+1}$$

If $R = 0$, set $k = j - 1$, $Y^*_{k+1} = U_{k+1}$, $t = j+s$. Stop.

If $R \geq L$, set $k = j$, $Y^*_{k+1} = R$, $t = k+s+1$. Stop.

If $R < L$, go to step 4.

Step 4: Go to step 3.

Remark 3: If $W - s\,L < 0$ at any stage, then the problem has no feasible solution.

Lemma5: The procedure detailed above will yield the optimal solution to problem P5.

Proof: Suppose, $\{Y_v\}$ is the optimal solution to P5. By Remark 2, the structure of the optimal solution is such that we can find values p and q such that

$$\sum_{i=1}^{p} U_i + Y_{p+1} + \sum_{i=p+2}^{q} L = W \qquad \text{... (17)}$$

where $L \leq Y_{p+1} \leq U_{p+1}$.

Note that $\sum_{i=p+2}^{q} L$ may not exist.

Now, we consider 3 cases depending upon whether $p > k$ or $p = k$ or $p < k$.

Case 1: $p > k$

This is not possible as by definition, k is the largest integer which satisfies (16).

Case 2: $p = k$

Then q cannot be less than t, as t is the smallest integer which satisfies (16). Hence $q \geq t$.

If $q > t$, then (16) - (17) gives

$$Y^*_{k+1} + (t-k-1) L - Y_{k+1} - (q-k-1) L = 0$$

i.e. $Y^*_{k+1} = Y_{k+1} + (q-k-1-t+k+1) L$

$$= Y_{k+1} + (q-t) L$$

Substituting for Y^*_{k+1}, the objective function value for the problem given by (16) is

$$\sum_{i=1}^{k} C_i U_i + C_{k+1} (Y_{k+1} + (q-t) L) + L \sum_{i=k+2}^{t} C_i$$

$$= \sum_{i=1}^{k} C_i U_i + C_{k+1} Y_{k+1} + L \sum_{i=k+2}^{q} C_i + L(\sum_{i=t+1}^{q} C_{k+1} - C_i)$$

$$< \sum_{i=1}^{k} C_i U_i + C_{k+1} Y_{k+1} + L \sum_{i=k+2}^{q} C_i$$

The last inequality is true as $C_t < C_j$ for $j>t$.

i.e. the solution given in (17) is not optimal.

Hence $q = t$ i.e. the procedure leads to optimal solution.

Case 3: $p < k$

There are two sub cases depending upon whether $q \leq t$ or $q > t$.

Case 3.1: $q \leq t$.

(16)-(17) gives

$$\sum_{i=p+1}^{k} U_i + Y^*_{k+1} - Y_{p+1} + L (t-k-1) - L (q-p-1) = 0$$

i.e. $$\sum_{i=p+2}^{k} U_i + (U_{p+1} - Y_{p+1}) + Y^*_{k+1} + L\,(t-q) - L\,(k-p) = 0$$

i.e. $$\sum_{i=p+2}^{k} (U_i - L) + (U_{p+1} - Y_{p+1}) + (Y^*_{k+1} - L) + L\,(t-q) = 0$$

But since $U_i > L$, $Y_{p+1} \le U_{p+1}$, $Y^*_{k+1} \ge L$, and $q \le t$,

the LHS is greater than 0, which is a contradiction.

Hence $q > t$.

Case 3.2: $q > t$

Now, (16) - (17) gives

$$\sum_{i=p+1}^{k} U_i + Y^*_{k+1} - Y_{p+1} + L\,(t-k-1) - L\,(q-p-1) = 0$$

i.e. $$\sum_{i=p+1}^{k} U_i + Y^*_{k+1} - Y_{p+1} - L\,(q-t) - L\,(k-p) = 0$$

i.e. $$(U_{p+1} - Y_{p+1}) + \sum_{i=p+2}^{k} (U_i - L) + (Y^*_{k+1} - L) = L\,(q-t) \qquad (18)$$

The optimum cost corresponding to (17) is

$$\sum_{i=1}^{p} C_i U_i + C_{p+1} Y_{p+1} + L \sum_{i=p+2}^{t} C_i + L \sum_{i=t+1}^{q} C_i$$

$$> \sum_{i=1}^{p} C_i U_i + C_{p+1} Y_{p+1} + L \sum_{i=p+2}^{t} C_i + L\,(q-t)\, C_t$$

$$= \sum_{i=1}^{p} C_i U_i + C_{p+1} Y_{p+1} + L \sum_{i=p+2}^{k} C_i + L\,\left(\sum_{i=k+1}^{t} C_i \right)$$

$$+ C_t \left\{ (U_{p+1} - Y_{p+1}) + \sum_{i=p+2}^{k} (U_i - L) + (Y^*_{k+1} - L) \right\}$$

(substituting for $L(q-t)$ from (18))

$$= \sum_{i=1}^{p} C_i U_i + (C_{p+1} Y_{p+1} + C_t (U_{p+1} - Y_{p+1})) +$$

$$\{ L \sum_{i=p+2}^{k} C_i + C_t (\sum_{i=p+2}^{k} (U_i - L)\} +$$

$$\{ L C_{k+1} + (Y^*_{k+1} - L) C_t \} + L \sum_{i=k+2}^{t} C_i$$

$$\geq \sum_{i=1}^{p} C_i U_i + C_{p+1} U_{p+1} + \sum_{i=p+2}^{k} C_i U_i + C_{k+1} Y^*_{k+1}$$

$$+ L \sum_{i=k+2}^{t} C_i$$

The last inequlity is true as $C_j > C_t$ for $j > t$,

$U_i > L$, $Y^*_{p+1} \leq U_{p+1}$ and $Y^*_{k+1} \geq L$.

This is the objective function corresponding to (16). This is not possible since (17) is optimal for P5. Hence Lemma 5 is proved. ■

If the theatres are numbered as per non increasing cost i.e. $C_1 \leq C_2 \leq \ldots. \leq C_N$, then it is easy to show by a similar procedure, that there exists an optimal solution to Problem P5 with the structure as given in Remark 2. The procedure detailed above will yield optimal solution to the problem.

6. CONCLUSION

The mathematical formulation for arriving at an optimal cinema schedule is complex but by nested Dynamic Programming , one can arrive at the optimal schedule easily. In a special case, the solutions to subproblems can be arrived at without using dynamic programming algorithm.

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