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**Cluster-Based Diffusion: Aggregate and
Disaggregate Level Modeling**

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Abstract

In this paper, we present a modeling approach for aggregate and disaggregate level models for cluster based diffusion of a new technology. For a homogenous population, the Bass (1969) model has been used extensively to predict the sales of newly introduced consumer durables. In comparison, little attention has been given to the modeling of the technology adoption by the industrial units present in disparate groups, called *clusters*. We study the pattern of diffusion of a new technology in a representative two-cluster situation. In the aggregate level modeling, we develop a model in which potential adopters of both clusters learn about the new technology from each other. Then, to focus on relatively micro-level phenomena, such as different propensities of imitation and innovation of firms within a cluster, we propose an agent based disaggregate model for cluster based diffusion of technology. In these disaggregate models, we capture the effects of heterogeneity and the inter-cluster and intra-cluster distances between the agents.

Our results highlight two major points: (i) both aggregate and disaggregate models are in agreement with each other in terms of their patterns, and (ii) both of the models exhibit a form which is consistent with the Bass model. Thus, consistent with the general theme of “why the Bass model fits without decision variables” (Bass, Krishnan and Jain 1994), we find that the Bass model, when extended appropriately, can be expected to work well in the cluster based technology diffusion situation also. This modeling approach can also be applied in the related contexts such as diffusion of practices (e.g., Quality certifications) within a multi-divisional organization or across various networked clusters.

[Key words: Technology diffusion, Bass Model, Agent-based simulation]

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1 Introduction

Innovation is required by an economy / firm to grow and gain competitive advantage in its products and services. This gain could come from process improvement or better product / service attributes, which are not easily replicable. The diffusion of innovation is the study of the process of the spread of new ideas and technology through cultures. Formally defined, the diffusion of an innovation is a “process by which an innovation is communicated through certain channels over time among the members of a social system.” (Rogers 1995). A number of studies in the past have examined the issues related to how an innovation spreads through an economy. For comprehensive review of such studies, the reader is referred to excellent survey papers by Mahajan, Muller and Bass (1990; 1995).

The innovation could be in the form of new technological products (e.g., mobile phones), foods like pizzas, music styles like opera, and ideas like democracy or feminism, and so on. The classical diffusion model in the marketing literature is the S-curve model of spreading innovations. This model has successfully been fitted to new product innovations in many industries (e.g., Gurbaxani 1990). It was also found that the diffusion speed is highly industry-specific and can be rather slow in the case of technologies (Loch and Huberman 1999; Geroski 2000). Diffusion of a new technology is the evolutionary process of replacement of an old technology by a newer one for solving similar problems or accomplishing similar or new objectives. The past literature offers several explanations for adoption of radically new technologies.

One approach to speed up the diffusion process of technologies is through developing local networks of similar firms located in geographical vicinity. These networks are known as *clusters* (Guilani 2002). High levels of clustering can lead to knowledge spillovers, opportunities for tacit knowledge exchange through sharing of human resources and hence faster adoption of productivity (or competitiveness) enhancing innovation. The cluster philosophy plays a vital role in bringing together the leading firms (usually the innovators) in a cluster with other imitator firms in the same cluster. Moreover, geographically separate clusters can also learn from each other, using a process which we term as *cross-imitation* in this paper. Although considerable amount

of research has been done in the area of diffusion, to the best of our knowledge, no previous study has examined the properties of cluster-based diffusion of a new technology. In this paper, we propose such a model by a representative two-cluster situation.

Most studies on innovation diffusion modeling are rooted in the work of Bass (1969). The Bass model formalizes the aggregate level of penetration of a new product emphasizing two processes of communication: (1) external influence via advertising and mass media, and (2) internal influence via word-of-mouth. The decision of a consumer is described as the probability to adopt the new product during time and it is assumed to depend linearly on these two forces. The first force is not related to previous adopters and it represents the external influence of mass media; the other force is related to the number of previous adopters and it represents the internal influence of word-of mouth. The model is able to represent a cumulative S curve of adopters and the fast growth is generated by the social interaction between early and late adopters (Rogers, 1995).

From the perspective of the current research, it would be interesting to examine whether the Bass (1969) model retains its various properties when extended in the technology diffusion domain. In the current paper, this investigation is performed at two levels: (i) the Bass model is extended in the direction of cluster based technology diffusion using an aggregate level approach, and (ii) its comparison is provided with a disaggregate-level model. The aggregate-level model represents the technology adoption process at an overall of a homogenous population. The disaggregate-level model focuses at the technology adoption process of an individual adopter in the population. At various points in time, for a disaggregate model, an integrative calculation can readily provide the aggregate level adoption statistic.

Though some recent studies have addressed issues related to technology diffusion (e.g., Ganesh and Kumar 1996; Baptista 2000; Geroski 2000; Delre et al. 2006), two issues specific to the cluster-based view appear to have scope for further scrutiny, namely, imitation across clusters and heterogeneity of firms within clusters. To gain competitive advantage, firms develop social networks and alliances with their customers,

suppliers, and even competitors. In doing so, the firms (or agents) of one region (say, a country) are involved in activities like export, import, foreign direct investments, etc., through which the information about new technologies spreads from one region to another. At a relatively micro-level, the same can be said of interactions between clusters as well. This suggests that often an agent in one cluster learns of and adopts a new technology through the agents in another cluster. This paper presents an extension of the Bass model by taking into account the *cross-imitation effects* in diffusion of innovations in clusters.

The Bass model assumes homogeneity in the population in at least two different respects: with respect to characteristics of members of the population and with respect to the information diffusion mechanism. In the former sense, all members have the same probability to adopt the new product at the time defined by a stochastic allocation to various timing classes. In the latter sense, all members are exposed homogeneously to external influence (advertising) and to internal influence (word-of-mouth effect from the previous adopters). Manfredi, Bonaccorsi, and Secchi (1998) call the former behavioral homogeneity and the latter social homogeneity. This paper presents an agent based simulation approach to incorporate *heterogeneity* in cluster-based diffusion process.

The modeling approach discussed can also be applied in the related contexts such as diffusion of practices or quality standards within a multi-divisional organization or across various networked clusters. Although considerable amount of research has been done in the area of diffusion of standards, scant research exists in the area of diffusion of practices within an organization. Dahl and Hansen (2006) examine the importance of size, region and external pressures in the process of diffusion of standards. They report insights gained into the circumstances present when organizations adopt standards by studying the diffusion of the Common Language Standard (CLS). Their theoretical framework highlights the empirical phenomenon that standards occasionally spread extremely rapidly to some - but not all - organizations within the same field. Empirical evidence from quantitative surveys of civil servants and elected officials in Danish municipalities is used to analyze the field and organizational levels. The levels of external pressure and organizational resources are important in order to understand why some

municipal organizations have adopted the CLS whereas others have not. They find that the initial source of the standard as well as regional pressure (i.e., network effect) play a strong role in the diffusion process. Similarly, Albuquerque, Bronnenberg and Corbett (2007) study the global diffusion of ISO 9000 and ISO 14000 certification using a network diffusion framework.

To summarise, the objectives of this paper are as follows. We first develop a conceptual framework and an aggregate model for cluster based diffusion of technological innovation. We next examine the empirical properties of the proposed diffusion model to compare it with the Bass (1969) model. Then, in order to contrast it with a disaggregate-level model, we develop an agent based model for cluster based diffusion process.

The rest of this paper is organized as follows. In the next section, building on background literature, we present a conceptual framework for our proposed model for the cluster based diffusion. The formulation of the various corresponding models is presented in Section 3. In Section 4, the results from a simulation analyses are presented and discussed. Section 5 concludes with some suggestions for extending this work.

2 Conceptual Framework

The internal influence in the decision making of a potential adopter in a cluster or customer in a market is well understood and documented (Mansfield 1961; Bass 1969; Mahajan et. al. 1990). Delre et al. (2006) propose that a potential adopter of an innovation or product can decide to adopt on the basis of mass media communication effect and word of mouth effect. The word of mouth effect depends on the level of interaction between the potential customer and previous adopters in the market. These authors only consider adopters from the same population, or single cluster. As we deal with the problem of diffusion in multiple (two) clusters, we propose that if a potential adopter interacts with a previous adopter of another cluster, then his timing to adopt the innovation can be influenced by this interaction (refer also Keely 2003; Mort 1991;

Geroski and Mazzucato 2001). We define two clusters each having firms with attributes such as innovation, imitation and cross-imitation abilities. Internal firm dependent variables influence innovation ability while interaction of firm within the same cluster influences imitation ability. The interaction of firms from one cluster with those of the other cluster influences their cross – imitation ability.

2.1 Social Network

A useful technique for studying information processing in this scenario is to construct a network that models the flow of information (Mitchell 1999). In our framework, the agents in our two clusters of focus are connected by a communication network. The two clusters are distinct geographically and do not share any common boundaries. This social network consists of two things – nodes and link between the nodes. Nodes indicate the agents in social networks which comprise of firms within a cluster, who are potential adopters of technology innovation. The links connecting the nodes indicating the information transfer channels between the nodes, through which information reaches from one agent to another agent (firm) and firms are made aware of the new innovations in the technology of interest. In this social networking framework, an agent has two types of personal networks: first, it is connected to the other agents in its own cluster, and second, it is connected to the agents in the other geographically distinct cluster as shown in Figure 1. Mort (1991) defines this cross boundary interaction by a similar concept called the Percolation theory.

Social interactions in this system are of two types: proximal contacts among agents of the same network (in the cluster to which the agent belongs) and weak ties interactions with individuals belonging to different networks or to geographically distinct clusters (Goldenberg et al., 2001). In our framework, there exist weak interaction ties among the agents of two clusters as shown in Figure 1.

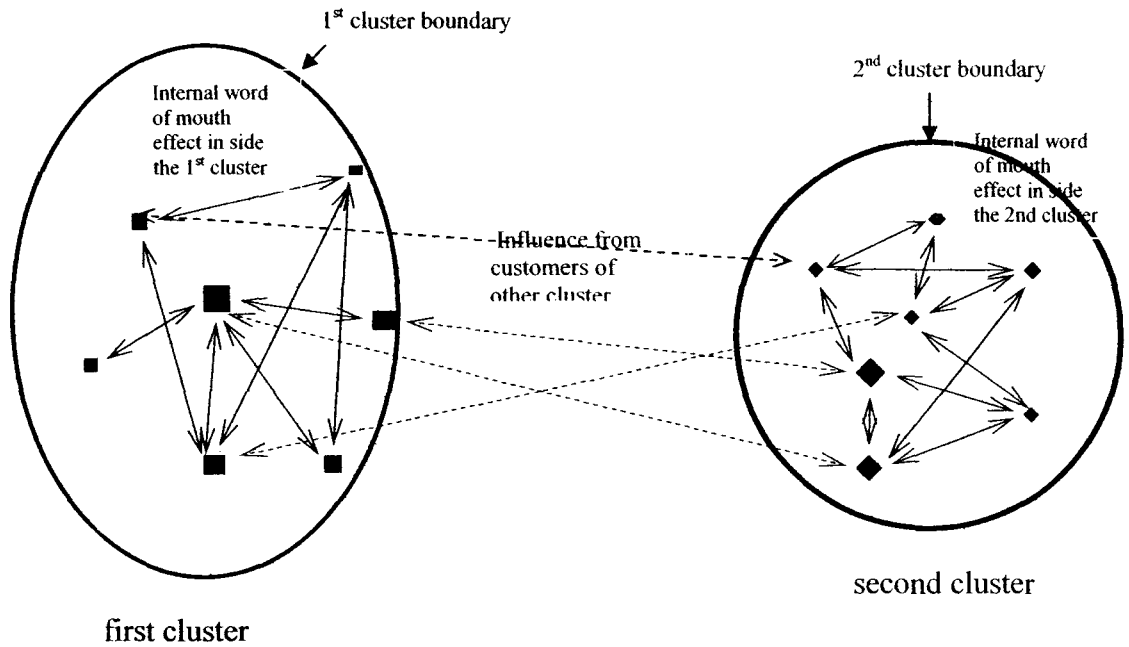


Figure 1: Communication Network

2.2 Aggregate Diffusion: Extended Bass (1969) Framework

In our aggregate diffusion model for two clusters, we develop the extended Bass's model, as shown in the schematic diagram in Figure 2. Through this model, we are trying to capture the spread of an innovation due to three forces, as described above: (i) innovation coefficient (α), (ii) imitation coefficient (β), and (iii) cross-imitation coefficient (γ). As shown in Figure 2, there is a population of potential adopters in a cluster, out of which a fraction is classified as potential innovators, while the remaining fraction is classified as potential imitators. The potential innovators become adopters following the innovator route at the rate of α in every time period. Two types of influences affect the potential imitators: a word-of-mouth or imitation effect (β) from within the same cluster, and an external influence effect due to interaction with the customers of the other cluster (γ)

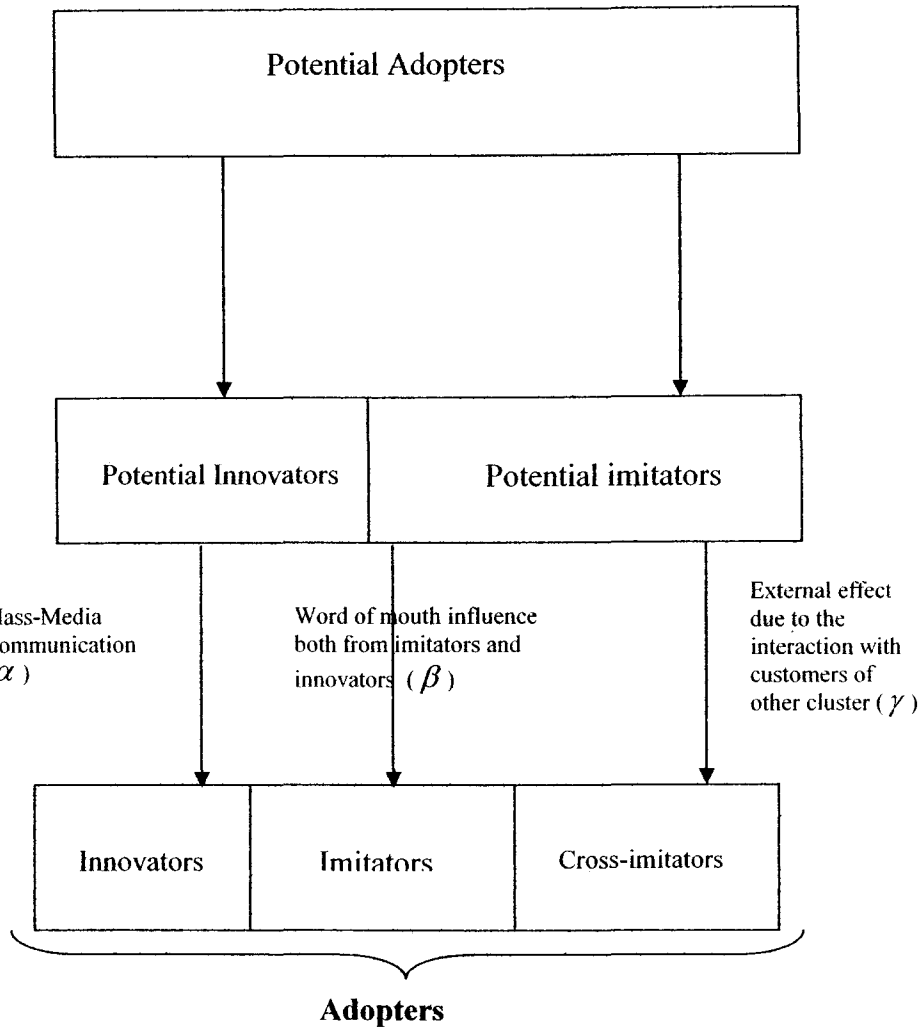


Figure 2: Schematic Diagram of Cluster Aggregate Diffusion Model

2.3 Disaggregate Cluster Based Diffusion

Similar to Delre et al. (2006), we formulate our disaggregate model of cluster based diffusion as a contagious epidemic in which information about the new product gets transmitted among the agents of the population like a virus that is transmitted in a computer network. The epidemic models are mostly divided in two categories: (i) SIS (Susceptible, Infected, Susceptible) and (ii) SIR (Susceptible, Infected, Removed). The former assumes that nodes are initially susceptible and they become infected with probability ν if they are directly linked with one or more infected nodes. Then the infected node recovers and becomes susceptible again with probability δ . When $\delta = 0$,

infected nodes cannot recover and the SIS model is converted into a SI (Susceptible, Infected) model. In the latter the same dynamics are assumed but once the node is infected, it just dies with probability γ , and it never recovers. For social and marketing situations, we focus on SIS and SI models because these are more relevant. Once an agent adopts a product, she is not removed from the market; on the contrary, her decision of adopting affects other consumers. In the beginning of the spreading process, SIS model assumes that the diffusion of disease infects only few nodes. The direct neighbors of these infected nodes become infected with probability ν . The study with random graphing technique shows that if $\lambda = \frac{\nu}{\delta}$ overcomes a given threshold λ_c , then diffusion speeds up, else the diffusion process dies out. Also, not all of the agents in the network may be involved in the diffusion process.

To define rules for information exchange and its impact related to technology dissemination and adoption in the two clusters, we make the following assumptions:

- 1) Each agent in a cluster has different diffusion parameters, whose values are randomly distributed .
- 2) Only an agent who has adopted the technology can influence the adoption decision of a potential adopter, whether through imitation or cross imitation.
- 3) Once an agent has adopted the technology, it cannot adopt it for a second time. This is different from the models in the marketing literature where multiple adoptions (repeat purchases) are allowed.

Based on the above rules, we formulated two different models to test different aspects of cluster diffusion, as detailed in the next section.

3 Cluster Based Diffusion Models

3.1 Aggregate Model of technology diffusion in two clusters

In this paper, we focus on the diffusion of technology in two clusters. We extend the frame work of Bass (1969) to develop the model. Similar to the Bass (1969) model, our aggregate cluster based technology diffusion model derives from a hazard function (the

probability that an adoption will occur at time t given that it has not occurred before time point t). Thus, the likelihood of purchase at time t given that no purchase has been made is $\frac{f(t)}{(1-F(t))}$ where $F(t)$ is cumulative number of adoption up to time t and $f(t) = \frac{dF(t)}{dt}$ is the instantaneous adoption at time t . It is well-documented in the literature that

the hazard function gives rise to the following differential equation:

$$dX/dt = (K-X)(\alpha + \beta X) \dots\dots\dots(1)$$

Here K denotes the size of the population and X denotes the cumulative adoption till time t . Previous researchers (e.g., Mahajan, Muller and Bass 1990) have shown that this equation gives closed form solution with several nice properties. This model can be extended to a cluster situation as follows.

Consider two clusters (markets or countries) comprising M and N firms respectively. Let X be the cumulative number of adopters in first cluster that has adopted the new technology by time t and Y be the cumulative number of adopters in second cluster that has adopted the new technology by time t . The rate of the adoption of new technology depends on the following:

- 1) Coefficient of innovation (α): accounts for the innovativeness of the potential adopters in a cluster.
- 2) Coefficient of imitation (β): accounts for the learning about the new technology, from the previous adopters in a cluster.
- 3) Coefficient of cross-imitation (γ): accounts for the learning about the new technology from the previous adopter of other clusters.
- 4) The potential adopters who have not adopted yet.

Mathematically, the rates of adoption of new technology in the two clusters can be shown as:

$$\frac{dX}{dt} = (M - X)(\alpha_1 + \beta_1 X + \gamma_1 Y) \dots\dots\dots(2)$$

$$\frac{dY}{dt} = (N - Y)(\alpha_2 + \beta_2 Y + \gamma_2 X) \dots\dots\dots(3)$$

where,

$\alpha_1, \beta_1, \gamma_1 \rightarrow$ are coefficients of innovation, imitation and cross imitation of first cluster, respectively.

$\alpha_2, \beta_2, \gamma_2 \rightarrow$ are coefficients of innovation, imitation and cross imitation of second cluster, respectively.

3.2 Properties of the Model

Equations (2) and (3) represent a nonlinear system of equations, for which no explicit analytical solution is available. It is similar to Lotka – Volterra competitive model for which Morris and Pratt (2003) have done statistical analysis. In this paper, we have solved equations (2) and (3) using the *Matlab* software package as well as simulation in C language to arrive at numerical solutions. The results are presented in next section.

3.3. Disaggregate model for cluster based diffusion model

To develop disaggregate model of cluster based technology diffusion, we use Agent Based Modeling (ABM) approach. Agent based and other computer based simulation methods have been used increasingly to study the social system since 1990s and have become a powerful mean of understanding social behavior and its dynamics. Agent based simulation involves a rule based interaction of individual entities called “Agents”. These rules are local to the environment in which the agent is interacting with other agents. Agent based modeling enables researchers to test and develop theories in a simple way that might not be possible by analytical and experimental techniques. For example, a researcher can study the effect of “fear” and “emotions” on the living standards of a population by imposing appropriate rules (Bonabeau, 2002).

To model the above-mentioned diffusion process, we consider two clusters. Adopters in these two populations are considered as agents. An agent can adopt the new technology by getting influenced by any of the three diffusion processes or by the combined effect of the three. Let these two clusters have M and N potential adopters (or agents), and an agent in each cluster is denoted with subscripts i and j respectively. The main question of concern is when an agent will adopt the new technology. Towards this end, we denote p_i (for agent from cluster of population M) and p_j (for agent from cluster of population N) as the probabilities that an agent adopts the technology in the two

clusters at any point in time. The probability of adopting the technology by an agent depends on three factors.

1. The Innovation effect: α_i and α_j are taken as coefficients of innovation for the clusters of size M and N respectively for i th and j th agent.
2. The Imitation effect: β_i and β_j are taken as the coefficients of imitation for i th agent in cluster M and for j th in cluster N , respectively. Let m_k be the no. of agents who have already adopted the product by the k th iteration. Therefore, the total imitation effect which affects i th agent to adopt the technology is $\beta_i \frac{m_k}{M}$. Similarly, the imitation effect for the j th agent in the second cluster is $\beta_j \frac{n_k}{N}$, where n_k is the number of agents who have already adopted the new technology in the second cluster.
3. The Cross – Imitation effect: γ_i and γ_j to be the coefficients of cross-imitation for i th and j th agents in the two clusters. There are n_k agents in the cluster of size N who have already adopted the new technology at the end of k iterations. Then, the cross-imitation effect in the first cluster of size is $\gamma_i \frac{n_k}{N}$. Similarly, the cross-imitation effect for an agent in the second cluster is $\gamma_j \frac{m_k}{M}$.

From the above explanation, we can express p_i and p_j in following equations.

$$p_i = \alpha_i + \beta_i \frac{m_k}{M} + \gamma_i \frac{n_k}{N} \dots\dots\dots(4)$$

$$p_j = \alpha_j + \beta_j \frac{n_k}{N} + \gamma_j \frac{m_k}{M} \dots\dots\dots(5)$$

Since we are modeling each individual agent as a different entity, the diffusion parameters in Equations (4) and (5) are randomly distributed between zero and specific values of the diffusion parameters of the clusters. For example, for i th agent in first cluster α_i is randomly distributed between 0 and α_1 , similarly β_i and γ_i are randomly distributed between 0 and β_1 , and 0 and γ_1 , respectively.

The decision to adopt the new technology depends on the utility threshold of an agent. If the probability of the adoption of the new technology of an agent exceeds this utility threshold for that agent, then the agent decides to adopt. In this way the agents become infected to the adoption (borrowing the terminology from the epidemic models). Let the utility thresholds of i_{th} and j_{th} agents in the first and second cluster be r_i and r_j respectively. Now the i_{th} and j_{th} agents in first and second cluster adopts the new technology if and only if $p_i > r_i$ and $p_j > r_j$, respectively.

3.4 Distance Model

In this model we examine the effect of the distance between the agents on the decision making of an agent to adopt the product. As suggested by Keller (2002), adjacent countries in a continent learnt more about the new technology than countries of the other continents. Hence, we propose that the imitation effect also depends on the distance between the agents. An agent who is having a smaller distance from a previous adopter learns more about the new technology than an agent who has a large distance from a previous adopter. First we formulate a model for a single cluster.

In the case of a single cluster, the probability that agent i adopts the new technology is as follows

$$p_i = \alpha_i + \sum_j \frac{\beta_i}{x_{ij}} g_j c_j, \text{ and } i \neq j \dots\dots\dots(6)$$

where,

p_i = probability of adoption of agent i

α_i = coefficient of innovation of agent i

β_i = coefficient of imitation of agent i

x_{ij} = distance between agent i and j ,

g_j = is a coefficient which keeps track of the neighborhood of i_{th} agent. It is defined as follows:

$g_j \rightarrow 1$ if j th agent is in neighborhood of i th agent

0 else

c_j = is coefficient which accounts for whether j th agent in the neighborhood of i th agent has adopted the technology or not. It is defined as follows:

$c_j \rightarrow 1$, if j th agent in the neighborhood of i th agent adopted the technology
 0, else

In the case of two clusters, an agent learns about the new technology from two effects: imitation effect from agent of its own cluster and cross-imitation effect from agents of the other cluster. Imitation effect on an agent from the previous adopters depends on its distance from the previous adopter in its own cluster. Cross-imitation effect on an agent depends on the distance between the agent and the previous adopter in the other cluster. The model is formulated as given below.

$$p_i = \alpha_i + \sum_l \left(\frac{\beta_l}{d_{il}} g_l c_l \right) \frac{m_k}{M} + \sum_j \left(\frac{\gamma_j}{d_{ij}} g_j c_j \right) \frac{n_k}{N}, \text{ for all } l \neq i \dots \dots \dots (7)$$

where all the terms have there general meaning in the earlier agent Based model for two clusters, except

$d_{il} \rightarrow$ is the intra-cluster distance between i th and l th agent in first cluster

$d_{ij} \rightarrow$ is inter-cluster distance between agent i of first cluster and agent j of the other cluster

$\sum_l \left(\frac{\beta_l}{d_{il}} g_l c_l \right)$ is the total Imitation effect or social pressure on the agent i from all l agents

who have already adopted the new technology in its own cluster. $\sum_j \left(\frac{\gamma_j}{d_{ij}} g_j c_j \right)$ is the total

cross-imitation effect on agent i from all agents j of other cluster, who have already adopted the technology. The effect of neighborhood of previous adopter in second cluster can be captured by g_j (neighborhood coefficient for j th agent if it lies in radius of inter-cluster of agent i in first cluster) and c_j (coefficient for j th agent in second cluster if it has already adopted). It is noted here that the presence of the distance terms in the denominators of Equation (7) produces the desired effect of increasing the probability of adoption in case of the adopting agents being in closer vicinity of an agent. An agent in the first cluster will adopt the technology if and only if its probability of adoption p_i is greater than its utility threshold r_i i.e. $p_i > r_i$. In the same way, an agent in second cluster

will adopt the technology if its probability of adoption p_j is greater than its utility threshold, r_j . p_j is defined as

$$p_j = \alpha_j + \sum_i \left(\frac{\beta_j}{d_{ji}} g_{i,c_i} \right) \frac{n_k}{N} + \sum_i \left(\frac{\gamma_j}{d_{ji}} g_{i,c_i} \right) \frac{m_k}{M}, \text{ for } j \neq l \dots\dots\dots(8)$$

4 Results and Discussion

In this section, we present results of the models presented in the previous section. In Section 4.1, we present the results of the aggregate model (solved in *Matlab* as well as a C program) to demonstrate how relative values of α , β and γ in the two clusters affect the diffusion process. In Section 4.2, we present the results of the disaggregate model by taking into account the heterogeneity of firm within a cluster (i.e., α , β and γ coefficients vary for each firm in the cluster). We also examine the role of distance between firms in the diffusion process.

4.1 Aggregate Model

The model as formulated in Equations 2 and 3 in the previous section is solved using Matlab software and the results are cross checked with a C language program. The values of diffusion parameters are taken from Ganesh and Kumar (1996), Helson et al. (1993), Gatignon et al. (1989) and Bass (1969) except the size of clusters. Ganesh and Kumar (1996) estimated the values of coefficient of innovation less than 0.1 and between -0.39 to 0.06 for nine lag countries. The value of coefficient of imitation is between 0.578 to 0.903 for Bass Model and 0.214 to 0.933 for lag countries. The value of cross-imitation coefficient (they term this the learning coefficient) vary between 0.0078 and 1.07. Gatignon et al. (1989) estimated the values of coefficient of innovation for consumer durables less than 0.1 and the value of coefficient of imitation up to 0.841. Helson et al (1993). estimated coefficient of innovation less than 0.01 and the value of coefficient of imitation between 0.256 and 0.728.

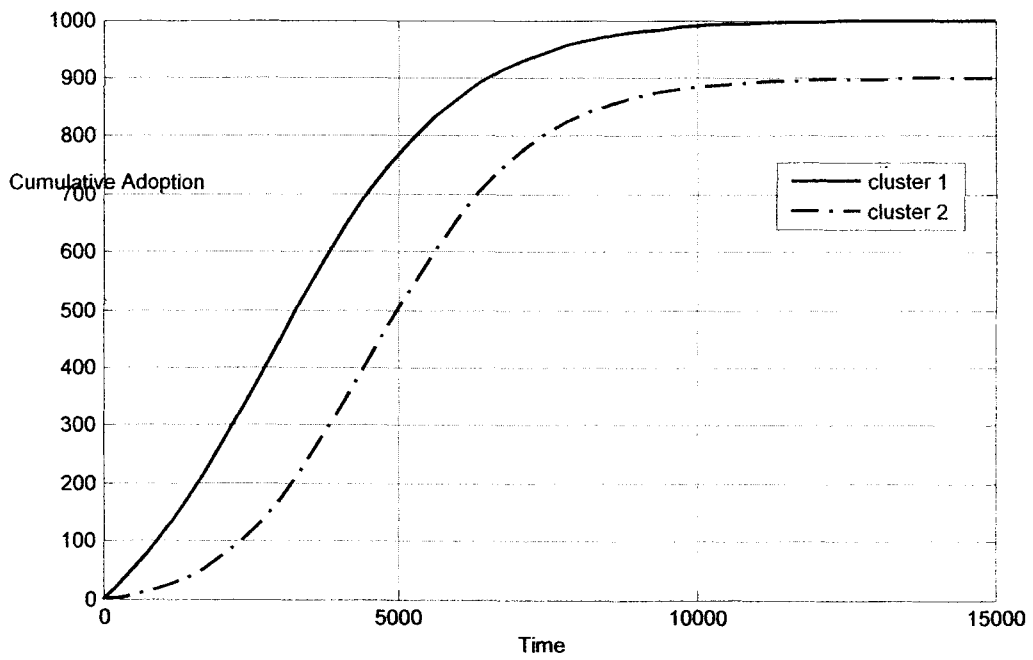
4.1.1 Main Results

From the above estimates, the following base values of the different parameters are chosen.

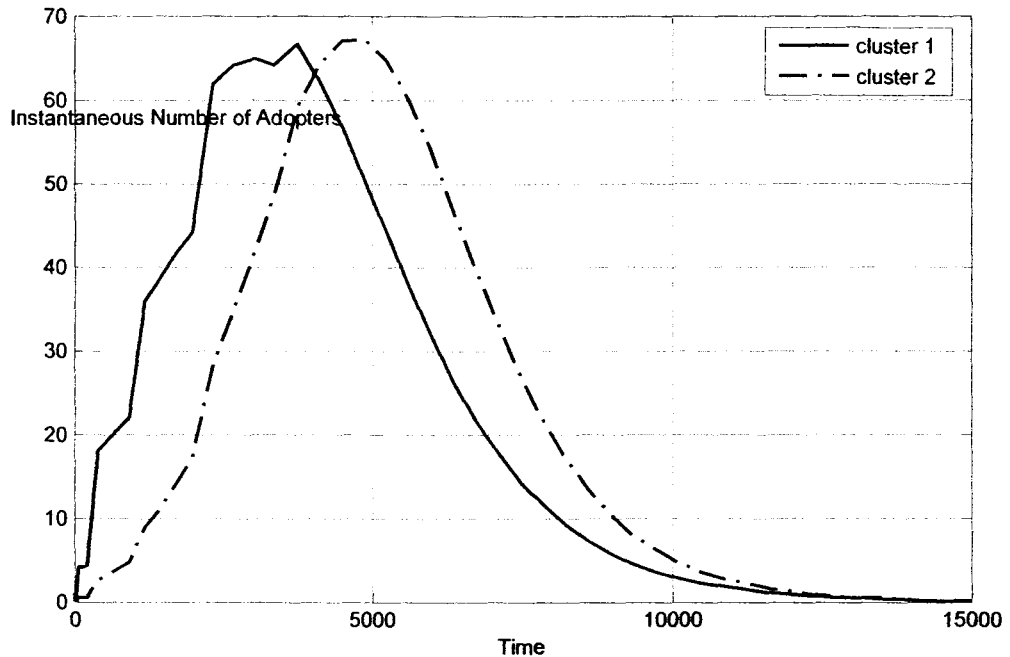
Table 1: Diffusion Parameter for Simulation

Cluster	Diffusion parameters	α	β	γ
1 st cluster (M =1000)		0.09	0.6	0.15
2 nd cluster (N=900)		0.01	0.55	0.1

Figure 3(a) and (b) summarizes the results obtained from the simulation using the parameters of Table 1.



(a)



(b)

Figure 3: (a) Cumulative and (b) Instantaneous Adoption of Technology in Two Clusters

It is clear from the Figure 1(a) that cumulative adoption curve of both clusters follows an S-curve, as predicted by the Bass model. Cluster 1 (size 1000) shows a faster adoption pattern of technology and the diffusion process completes in 12720 time points. It takes 13100 time points for cluster 2 for complete diffusion of the new technology. Diffusion is slow in cluster 2 due to lower values of all three diffusion parameters in comparison to cluster 1. The instantaneous adoption curve in Figure 1(b) depicts a bell-shape, as expected. The processes of diffusion of new technology starts rapidly in both clusters, reaches a maximum value (point of inflection) and then phases out. Instantaneous adoption follows a clear bell-shaped curve in Cluster 2, while it follows a near bell-shape in cluster 1. Interestingly, the maximum adoption reaches 67 firms despite the different diffusion parameters and cluster sizes in both clusters; however, the timing of maximum adoption is different. Maximum adoption occurs at time point 3723 in cluster while it occurs at time point 4473 in cluster 2.

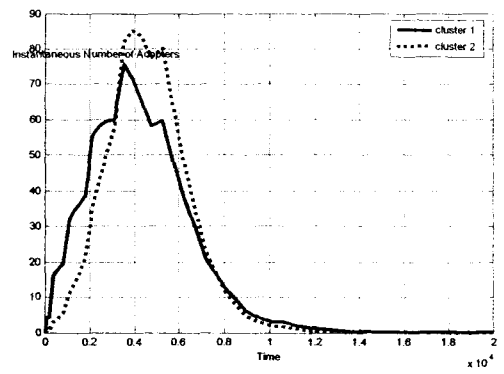
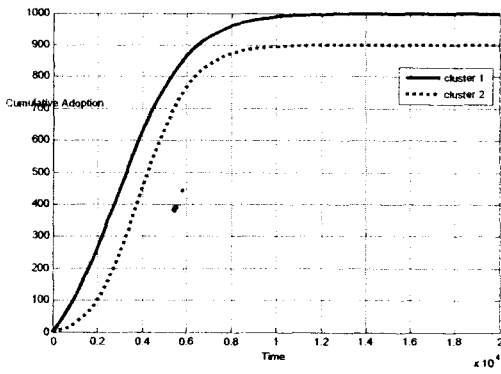
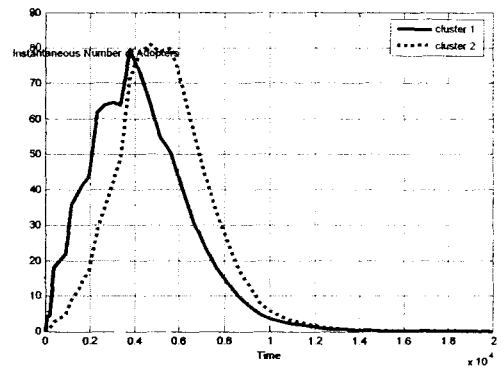
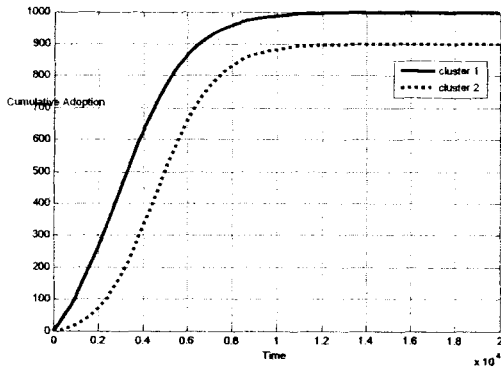
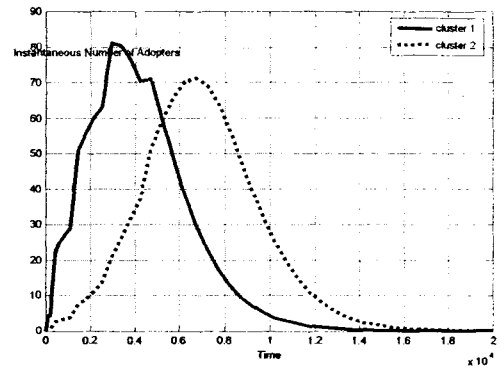
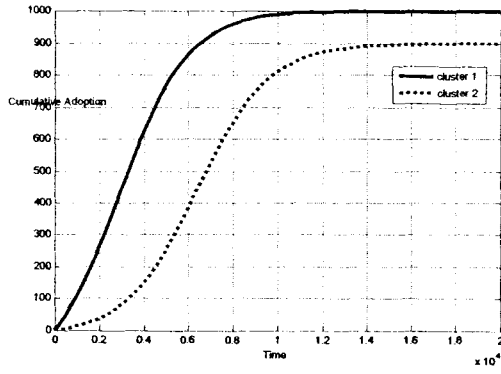
4.1.2 Variation in Diffusion Coefficients

In this section, we discuss various scenarios with different values of diffusion parameters. First we vary the coefficient of innovation for both clusters without any change in other parameter. After that, we change the coefficient of imitation for both clusters. Finally, we vary the pattern of diffusion process with changes in coefficient of cross – imitation. However, as the results of coefficients of innovation and imitation have already been discussed in the previous literature, in this section, we only present the results of variation in the coefficient of cross-imitation.

The effect of γ_1 and γ_2 on the diffusion of new technology in two clusters are shown in Figure 4. We chooses three values for γ_1 : 0.001 (low), 0.1 (medium), and 0.2 (high). The values taken for γ_2 are 0.001, 0.01, 0.1, 0.2, 0.3 and 0.4. (also listed in the table below). In this way, we investigate three sets comprising six cases of input data. The results shown in Figure 4 are for the low setting of γ_1 .

Table 2: Variations of γ_1 and γ_2

Scenarios	γ_1	γ_2
low γ_1 , vary γ_2	0.001	0.001, 0.01, 0.1, 0.2, 0.3 ,0.4
Medium γ_1 , vary γ_2	0.1	0.001, 0.01, 0.1, 0.2, 0.3 ,0.4
high γ_1 , vary γ_2	0.2	0.001, 0.01, 0.1, 0.2, 0.3 ,0.4



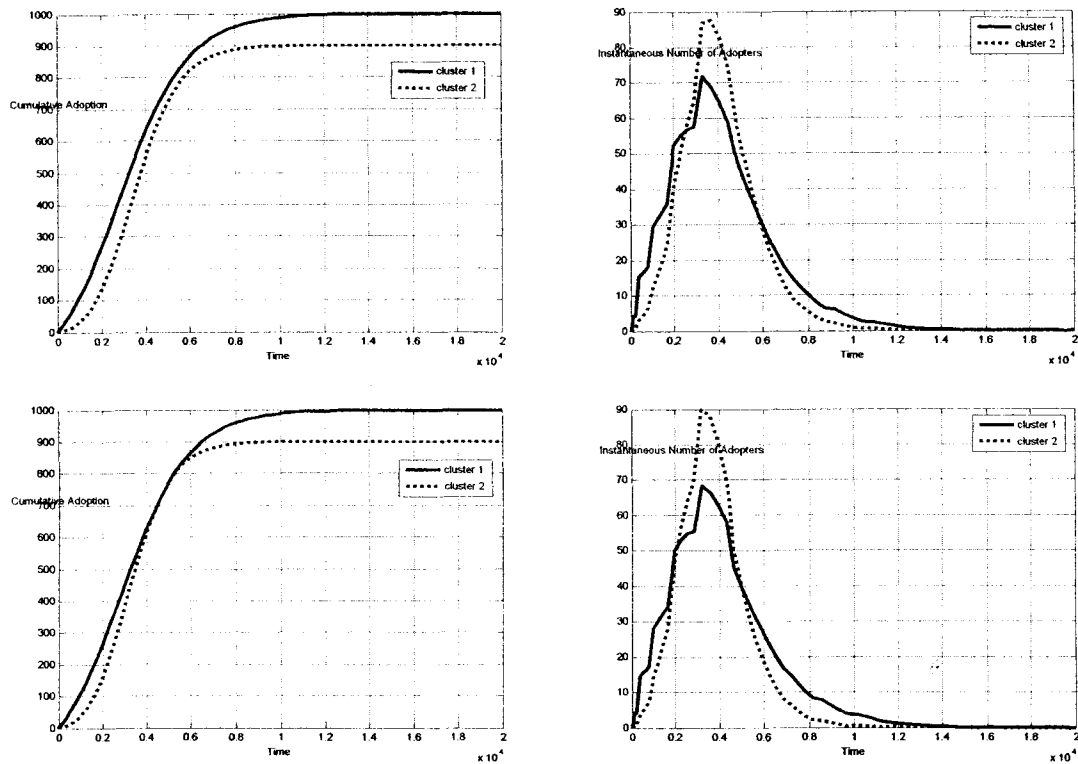


Figure 4: Cumulative and Instantaneous Adoption with $\gamma_1 = 0.001$ and Varying γ_2

In Figure 4, cumulative adoption curve follows the S-curve in all cases and all sets (low, medium and high γ_1). An interesting aspect about the cumulative curves is that, for a given value of γ_1 , as the value of γ_2 increases, the cumulative adoption curve of cluster 2 shifts towards the cumulative adoption curve of cluster 1. However we do not observe a cross-over of the curves till the setting ($\gamma_1 = 0.001$; $\gamma_2 = 0.4$), at which point we observe that the upper portion of cumulative curve for cluster overlaps with the cumulative curve of cluster 1. This led us to plot a special case with $\gamma_1 = 0.001$; $\gamma_2 = 0.5$, as shown in Figure 5. Here the cumulative adoption curve of cluster 2 crosses the cumulative curve of cluster 1 two times, once when the diffusion accelerates in cluster 2 and then when the diffusion is about to reach completion in cluster 2.

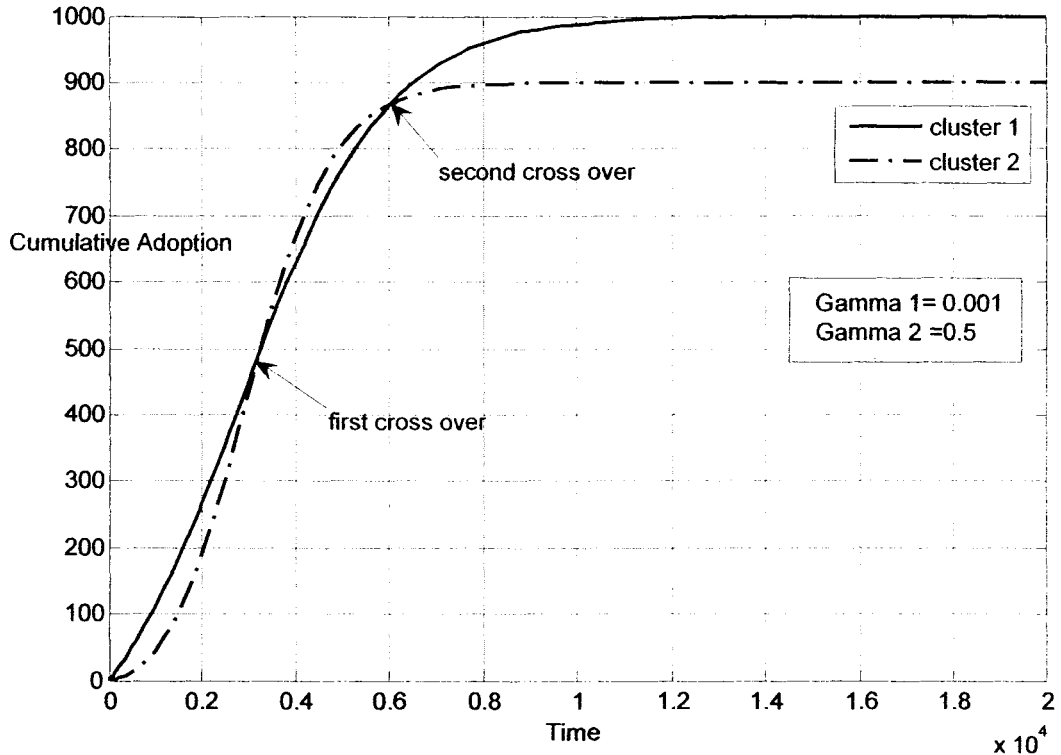


Figure 5: Two Cross-over Points for $\gamma_1 = 0.001$; $\gamma_2 = 0.5$

4.2 Results of Disaggregate Model for Cluster Based Technology Diffusion

The aggregate models describe homogeneous situations. The social environment (i.e., the population) in which the information about the new technology spreads is homogeneous in that (i) all the individuals are homogeneously exposed to the public sources of information, and (ii) all the individuals mix homogeneously, that is, they are homogeneously exposed to internal communications as well as external communications.

To study the personal characteristics of an individual of a cluster we developed four heterogeneous models of cluster based diffusion of technology. To solve the disaggregate models we used Agent Based Simulation approach. These models have been explained in the earlier sections. To solve the disaggregate model we coded the models in C programming language.

4.2.1 Result of Heterogeneous Model of Cluster Based Diffusion of Technology

In the formulation of this model, we assume that an agent or individual of a cluster learns about the new technology by the external source, the imitation from the cluster and

cross-imitation from the other cluster. Therefore, we supply distinct values of diffusion parameters for each agent or firm of both clusters. The assigning of the individual diffusion parameters of firm is done randomly. After using the randomly generated diffusion parameters of each agent or firm, the output obtained in terms of cumulative and instantaneous adoption is shown in Figures below.

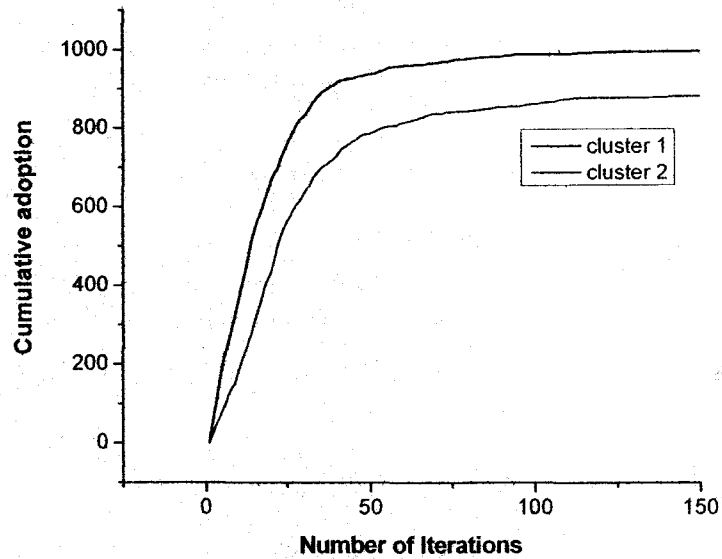


Figure 6: Cumulative Adoptions in Two Heterogeneous Clusters

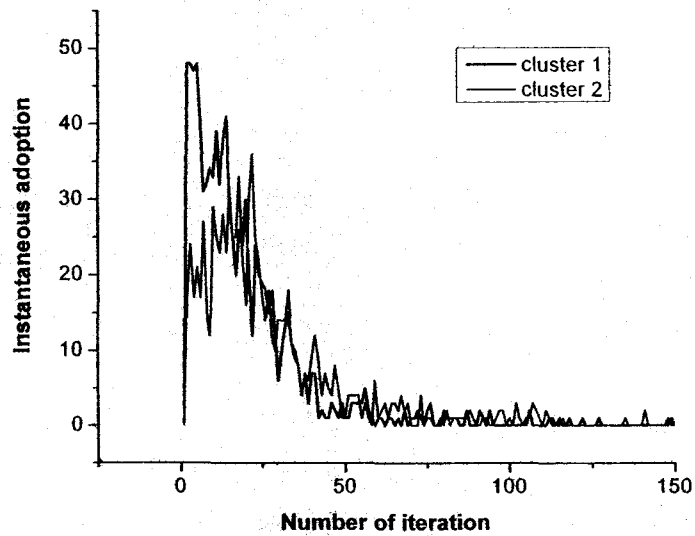


Figure 7: Instantaneous Adoptions in Two Heterogeneous Clusters

The cumulative adoption and instantaneous adoption curves in Figures 6 and 7 are slightly different from that of the aggregate results. First cumulative adoption curve for both cluster does not follow a strict S-curve. Second, as can be expected for an agent-based simulation approach, the instantaneous adoption does not show a very smooth pattern. However, the two types of curves approach the form of the curves obtained in aggregate results, lending support to the efficacy of Bass model in such situations.

4.2.2 Results of Distance Model for Single Cluster Case

We use the same values of diffusion parameters in the distance model of cluster based technology diffusion. The results of this analysis for a single cluster are shown in Figures 8 and 9. The cumulative adoption curve shown in Figure 8 follows an S-curve. Instantaneous adoption also tends towards a bell curve. Diffusion process was completed in the 14th iteration. The maximum adoption occurs in the 3rd iteration.

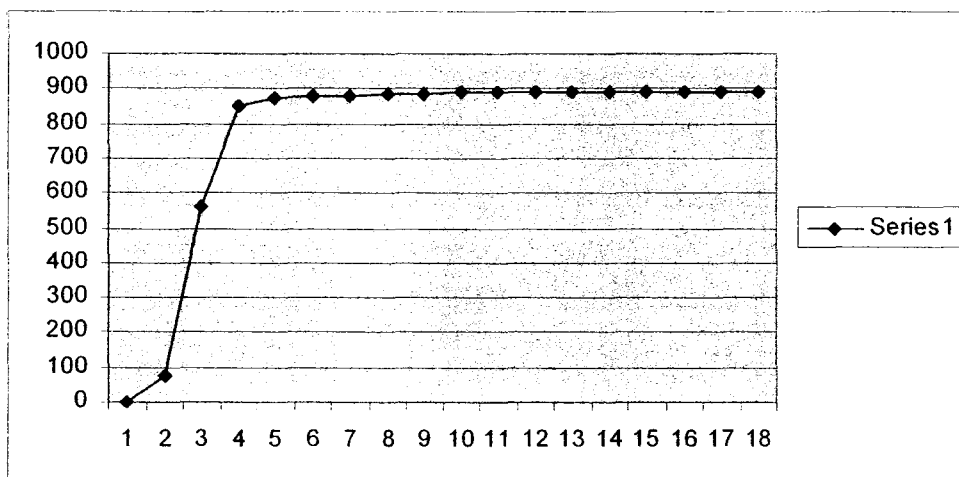


Figure 8: Cumulative Adoption Results of Distance Based Model

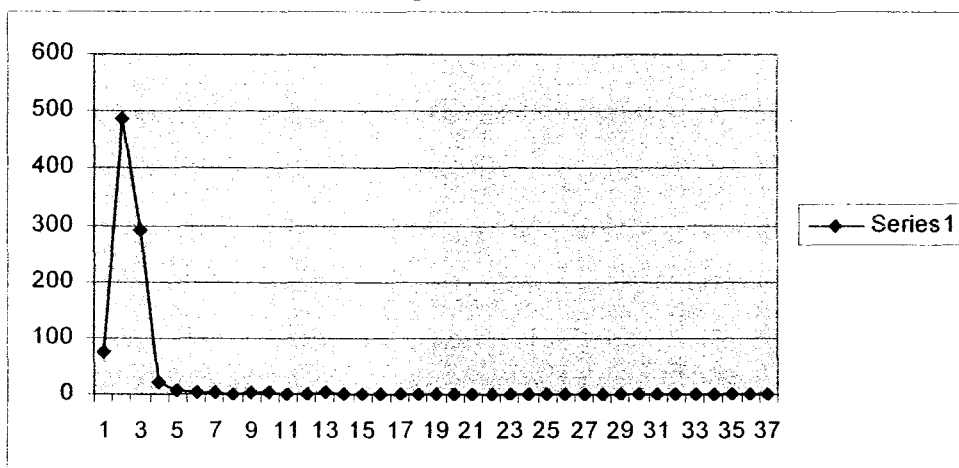


Figure 9: Instantaneous Adoption Results of Distance Based Model

4.2.3 Results of Distance Model for Two Clusters Case

Based on the encouraging results of distance based model for a single cluster, we ran a simulation for two clusters to examine the effect of distance on technology diffusion pattern. To run this simulation we used the same diffusion parameters as we used in the aggregate model. The results for the above parameters are shown in the figures below. It can be seen from the results that the distance based disaggregate model tends to mimic the results of the aggregate model.

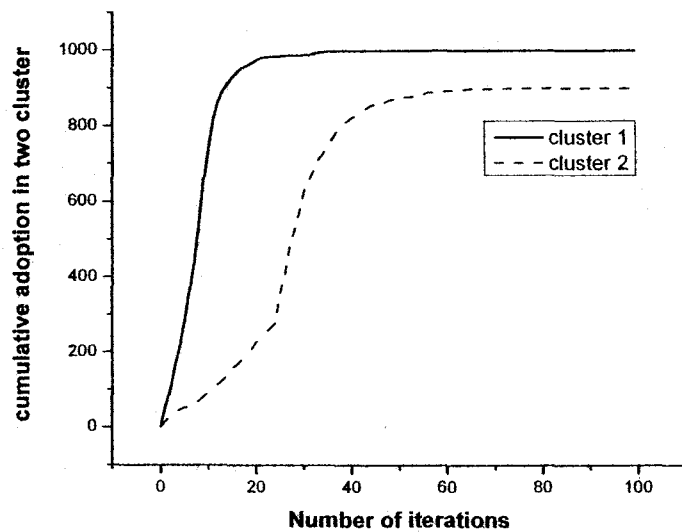


Figure 10: Cumulative Adoption in Two Clusters Distance Model

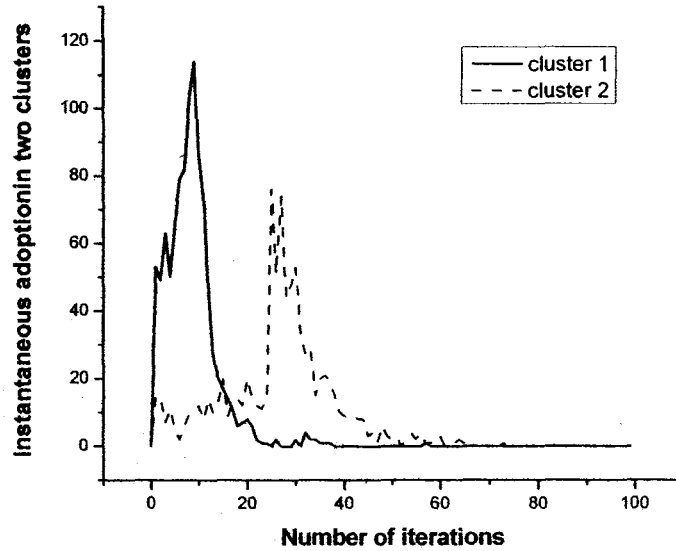


Figure 11: Instantaneous Adoptions in Two Clusters Distance Model

5 Discussion and Future Research

In this paper, we formulated aggregate and disaggregate models of cluster based technology diffusion and solved them numerically. The aggregate model appears as an extension of the classical Bass's (1969) diffusion model. The disaggregate model captures the heterogeneity of potential adopters and predicts technology adoption pattern in a population. The aggregate model for cluster based diffusion of technology consists of two non-linear differential equations. This system of equation has no analytic solution. We examined the diffusion pattern through extensive numerical analysis of the data. The disaggregate models were tested for diffusion parameters available in the literature and randomly generated data. Our results highlight two major points: (i) both aggregate and disaggregate models are in agreement with each other in terms of their patterns, and (ii) both of the models show the form consistent with the Bass model. Thus, consistent with the general theme of "why the Bass model fits without decision variables" (Bass, Krishnan and Jain 1994), we find that the Bass model can also be expected to work well in the cluster based technology diffusion situation. A major departure of our approach from the previous investigations towards this end is that we provide this verification at both aggregate and disaggregate levels.

We now discuss some future extensions of the current research. We assume a constant utility for the adopter that does not change over the time. From the supply side, due to continuous improvement in the technology by the R&D effort of the producer, the user friendliness of the technology for the potential adopters may change over time. From the demand side, potential adopters or firms of the technology may have different skill levels. These demand and supply side constraints define the utility of the technology for potential adopters and may effect the decision of the potential adopter to adopt the new technology. Thus, these effects can be incorporated in to the model. The model can be generalized for more than two clusters situation. Future research could also look at other forms of agent behaviours and the differences in their sizes.

Model for cluster based technology diffusion can equip a global marketing manager to formulate marketing strategy in international markets. It is well noted that firms interested in entering international markets can adopt either a simultaneous approach to enter multiple foreign markets (also called as sprinkler strategy) or a sequential approach in which the firms initially enters one or more lead markets and subsequently times enters into foreign markets in a phased manner (waterfall strategy). Which strategy should a firm adopt? In addition to the existing research (Kalish, Mahajan and Muller 1995), the cluster based diffusion models discussed in this paper can be used to provide some of these answers.

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