WORKING PAPER NO: 670

Foreign Reserves, Government Debt and Endogenous Risk Premium

Anubha Dhasmana
Assistant Professor of Economics & Social Sciences
Indian Institute of Management Bangalore
Bannerghatta Road, Bangalore – 5600 76
anubha.dhasmana@iimb.ac.in

Year of Publication – August 2022

Foreign Reserves, Government Debt and Endogenous Risk Premium

Abstract

Policy makers in Sub-Saharan Africa, as elsewhere, often need to find an operational way to assess reserve adequacy. This paper looks at the optimal level of foreign reserves for commodity exporting African countries using a small open endowment economy model. We assume that reserves provide insurance; allowing countries to smooth domestic absorption against the disruption induced by a large adverse price shock associated with a fall in output. Apart from the fall in output, price shocks are accompanied by a failure to roll over outstanding external short-term debt. For plausible values of model parameters, the model can account for the average level of reserve holdings by Sub-Saharan African countries during recent years. Actual value of optimal reserves is sensitive to the choice to benchmark parameters. We therefore do sensitivity analysis for our benchmark results towards the end.

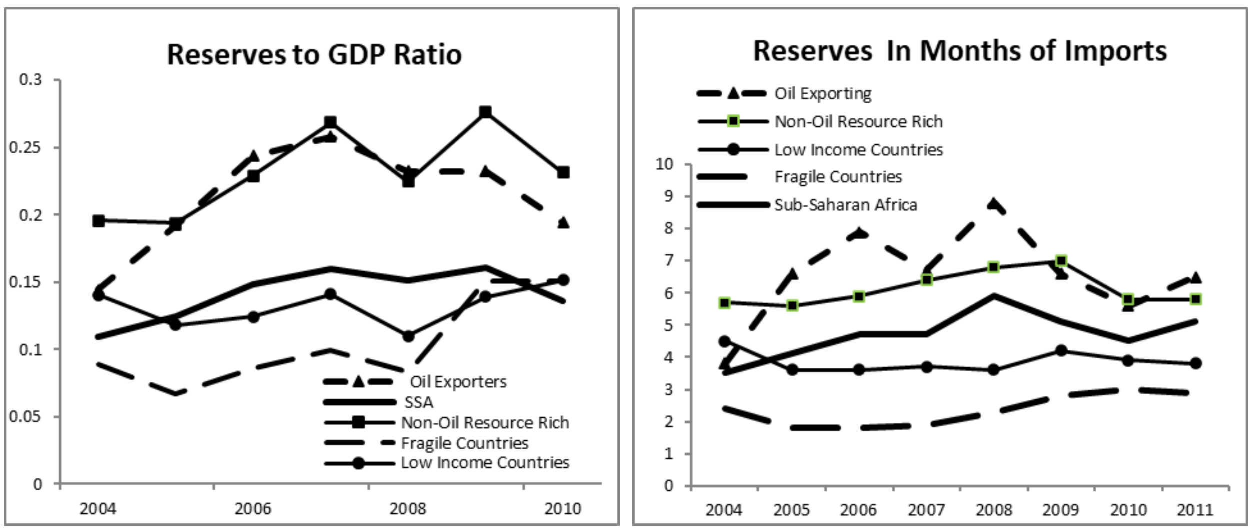
Keywords: Foreign Reserves, Africa

1. INTRODUCTION

Countries in Sub-Saharan Africa saw a significant increase in their reserve levels starting from mid 1990's; a phenomenon that gained further momentum beginning early 2003. This reflected insufficient initial reserve holdings, increasing openness of Sub-Saharan African economies, and a policy choice to build precautionary reserves to insure against balance of payment risks. The rising trend continued until the 2008 financial crisis when many of the commodity exporters in Africa were severely hit by a decline in the world demand and price levels for their key commodity exports. Figure [1] shows the movement of foreign reserves across different groups of Sub-Saharan African countries since 2004.

Figure [1]

Reserve Adequacy in Africa



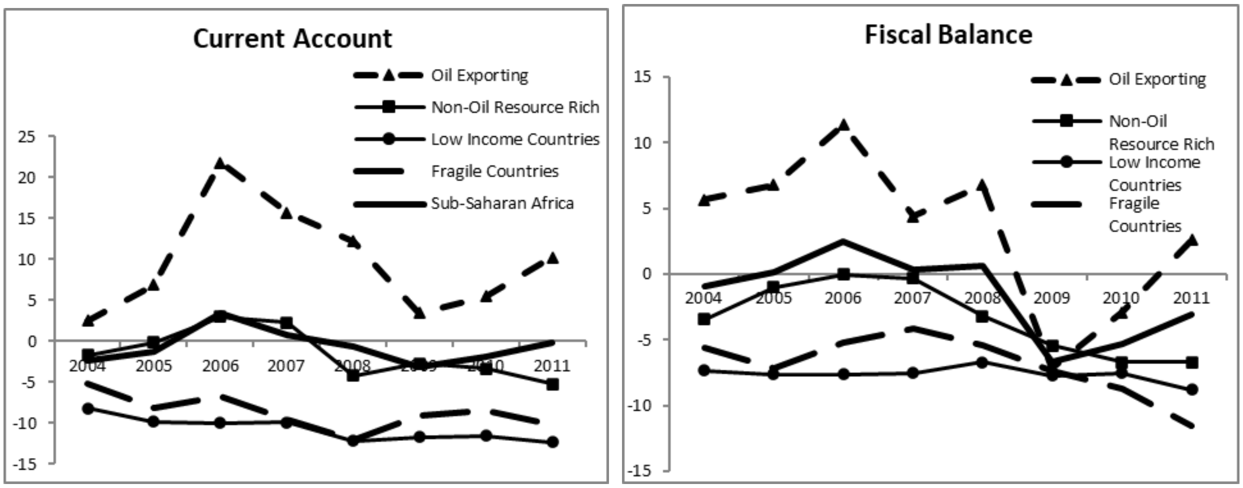
Source: REO

The oil exporting countries and, to a lesser extent, non-oil resource intensive countries accumulated significant amount of reserves with the help of rising commodity prices during 2004-2007. However, with the onset of the global financial crisis these countries saw a sharp decline in the prices for their key exports. The resultant decline in export revenues was reflected in the declining reserve levels for African commodity exporters.

Fall in commodity prices also adversely affected both Current Account and government budget deficits. Once again, oil importers and non-oil resource intensive countries saw the biggest adjustment in both (see Figure [2]). This episode has reignited the debate on adequacy of foreign reserves in commodity export dependent developing countries.

Figure [2]

Reserve Adequacy in Africa



Source: REO

Overall, the reserve to GDP ratio for Sub-Saharan Africa still remains below its peak of year 2007. However, there is enormous diversity across various groups within SSA countries in terms of their reserve holdings. While the oil and non-oil resource intensive countries carry reserves to cover up to six months of their imports (twice the prescribed rule of thumb of 3 months); the fragile counties carry less than the prescribed limit (in fact they had a reserve cover of less than two months of imports prior to the crisis). Whether this reflects a fundamental difference in the vulnerability of these groups to shocks or an inadequate reserve management policy is a matter of debate.

Table 1 compares foreign reserve levels of Sub-Saharan African countries with those of other regions around the world in terms of months of imports, as a share of short-term debt and as a percentage of GDP. While the first measure is a Current Account based measure of reserve adequacy the second one is a capital account based measure. The last measure is the consumer welfare based reserve adequacy measure derived by Jeanne and Ranciere (2011). Compared to the developing countries in Asia, North Africa and Middle East, Sub-Saharan African countries carry lower level of foreign exchange reserves when we look at these traditional measures of reserve adequacy. However, their reserve holdings are larger when compared to the low income and least developed countries in general.

Once again, experience of other regions provides only limited guidance for Sub-Saharan African countries about the adequate level of reserves in future since many of them have very different economic structure and face very different set of vulnerabilities compared to the Sub-Saharan African countries. Further, many countries may accumulate reserves as a side- effect of monetary and exchange rate policies, such as efforts to stem real exchange rate appreciation. In this case, the observed level of reserves is no benchmark for their adequacy.

Table [1]

Comparisons of International Reserves Across Regions, 1995-2010

	Reserves In Months of Imports				Short Term Debt to Reserves Ratio (%)				Reserves to GDP (%)			
	1995-2007	2008	2009	2010	1995-2007	2008	2009	2010	1995-2007	2008	2009	2010
Sub-Saharan Africa	7.25	5.96	6.78	5.33	94.6	29.2	21.5	21.75	10.0	15.1	16.1	13.6
Middle East And North Africa	22.4	24.7	26.3	24.5	22.9	6.42	5.46	5.33	19.3	38.8	44.8	43.4
East Asia & Pacific	8.78	13.5	18.6	16.5	32.2	11.4	11.5	13.8	17.7	30.5	35.0	34.7
Developing Only	8.82	16.1	22.1	19.5	32.3	11.4	11.5	13.8	22.1	38.3	43.5	43.3
Latin America	5.78	6.58	9.55	8.74	60.2	30	26.4	32.3	8.54	11.0	13.0	12.1
Developing Only	5.80	6.53	9.52	8.75	60.2	30	26.4	32.3	8.75	11.2	13.3	12.4
Low Income	4.14	3.76	5.19	4.82	73.9	27.9	17.1	18.9	7.36	8.68	11.2	10.5
Least Developed Economies	4.74	4.2	5.04	5.01	97.1	30.4	24.6	25.1	9.00	11.6	12.6	12.2

Source: World Development Indicators, Author's calculations

Overall, given the diverse and changing nature of African economies, traditional measures of reserve adequacy such as months of imports provide only limited guidance for SSA countries about the adequate level of reserves when faced with large exogenous shocks such as a fall in world commodity prices. The objective of this paper is therefore to provide a framework for analyzing reserve adequacy for commodity exporting African countries that takes in to account their unique economic structure and specific vulnerabilities.

We do this by using a small open economy model with two goods and a representative household consumer. Our small open economy is faced with exogenous terms of trade shocks that result in a loss of domestic output. During normal times private consumer can issue short term debt in order to smooth consumption. However, when there is an adverse price shock, there is no roll-over of external debt. Under these circumstances government can smooth domestic consumption during adverse price shocks by entering in to insurance contracts with foreign investors, or equivalently, by financing a stock of liquid reserves with contingent debt.

The model does not have a closed form solution. We therefore explore the quantitative implications of the model numerically using data on a sample of 35 African countries. The optimal level of reserves depends in an intuitive way on the probability and the size of the price shock, output cost of price shock and the opportunity cost of holding the reserves. For plausible parameter values our benchmark model predicts an optimal level of reserves equal to 16.5

percent of GDP. This is close to the average reserve holdings of Sub-Saharan Africa over last five years. Our benchmark model can therefore explain the reserve accumulation in Sub-Saharan Africa during recent years. We also present some extensions of our basic model, including one in which the opportunity cost of holding reserves is endogenous.

1.1 Comparing the Optimal level of reserves with the `rule-of-thumb' measures of reserve adequacy

One widely used `rule of thumb' measure for reserve adequacy is the Greenspan-Guidotti rule according to which countries should carry reserves equal to their short-term external debt. This reflects the vulnerability of the economy to shocks arising from the capital account. Greenspan-Guidotti rule is a natural benchmark of comparison for our model as the failure to roll over external short term debt is the key feature of our model. Comparing the optimal short-term-debt to reserves ratio from the benchmark model with the Greenspan-Guidotti rule we find that the optimal level of reserves can be above or below the latter depending upon the parameter values. For the benchmark parameter values the optimal reserves to short term debt ratio is in fact above the Greenspan-Guidotti rule of one.

Our paper relates to the recent literature on reserve adequacy that has taken the welfare of the representative agent as the criterion to maximize. These include Durdu (2007), Caballero and Panageas (2007) and Jeanne and Ranciere (2011).While our paper is most closely related to the last one, we extend their analysis to African commodity exporters by focusing on commodity price shocks that are accompanied by capital account shocks and also take in to account the potential impact of reserve levels on output loss due to adverse price shocks. Also, we assume that the government provides insurance against price shocks by issuing contingent bonds. This is different from the assumption in Jeanne and Ranciere (2011) where private sector buys insurance from foreign insurers.

Our paper contributes to the existing literature on reserve adequacy by taking in to account the impact of reserve holdings on output costs of adverse price shocks. Ignoring these effects can give us misleading results regarding reserve adequacy.
Paper is organized as follows: Section 2 presents empirical analysis of the role of reserves in the event of an adverse price shock. We try to measure the output cost of large adverse price shocks in the presence of threshold effects of reserves holdings. Section 3 presents the model for optimal reserves. Section 4 presents model calibration and simulation results including some extensions. Section 5 concludes.

2. Empirical Analysis

Higher level of reserves can mitigate the impact of adverse price shocks on consumer's welfare by allowing for a smoother consumption path and also by reducing the loss in income arising due to adverse price shocks. There are several potential channels through which reserves can cushion the impact of adverse price shocks on output or income. Lower exchange rate volatility and smoother current account adjustment are some of those. A few of these channels have been explored in the literature (see Aizenman and Riera-Crichton (2008) and Aizenman (2008)).

We try to measure the cushioning impact of reserve holding on the loss of output due to large adverse price shocks and incorporate that effect in the model. For this we use data on 35 African countries and find strong evidence that countries with lower level of reserves to short term debt ratio experience much higher loss in output as a result of adverse price shock. Further, this cushioning impact of reserves exhibits significant non-linearity. We also find evidence that this might happen due to smoother current account adjustment in countries with higher level of reserves relative to their short-term-debt.

Objective of this section is to empirically test the role of reserves in cushioning the impact of adverse commodity price shocks. As shown by Collier and Goderis (2009) and others, negative price shocks can have significant short-term impact on the output growth of commodity exporters. We want to test whether foreign reserves can reduce this loss in output due to negative price shocks. The main coefficient of interest for us is therefore an interaction term between adverse price shocks (construction of price shocks is explained in detail below) and the level of reserves carried by the country. Results of this empirical exercise are used to calibrate the theoretical model in section 4.

At the same time, in line with the literature, we try to take into account the long run relationship between commodity prices and real GDP growth. A large literature suggests that there is a 'resource curse': natural resource abundant countries tend to grow slower than resource scarce countries in general. However, whereas the resource curse literature predicts a negative effect of commodity price booms on growth, empirical studies by Deaton and Miller (1995) for Africa and Raddatz (2007) for low income countries find quite the contrary: commodity price booms significantly raise growth. The African growth acceleration coincident with the commodity boom that began in 2000 is clearly consistent with these findings. The sign of long-run relationship between these two variables is therefore a matter of empirical and theoretical investigation.

The other long-run variable in our model is world GDP per capita. We use deviations of world GDP per capita around its long run trend as a proxy for changes in world demand. One would expect real GDP per capita in African countries to have a positive relationship with this variable in the long-run.

We find strong evidence of a long run relationship between real GDP per capita, commodity price index and world GDP per capita using panel co-integration tests[1]. We therefore include

[1] Results from the Panel co-integration tests are available upon request.

these variables in our model using an error correction framework. Apart from these we include adverse commodity price shocks, inflation, volatility in terms of trade, foreign interest rate and a measure of government policy in our model as short run determinants of real GDP growth.

To capture commodity price movements we construct country specific commodity price index as suggested by Deaton and Miller (1995) and others. It is well known in the literature that aggregate terms of trade indices are unsuitable as proxies for commodity price movements as they include changes in non-commodity export prices. To address this issue we construct commodity export price indices for each country using commodity price data for forty-nine commodities. Construction of country specific commodity price index is explained in the Appendix.

Our main empirical analysis uses Pooled Mean Group estimator suggested by Pesaran et.al (1995). Pooled Mean Group estimator allows short run coefficients to vary across countries while keeping the long run coefficients same. It is a useful intermediate alternative between estimating separate regressions, which allows all coefficients to vary across groups and conventional fixed effects estimators, which assume that all coefficients and standard errors are the same.

2.1 Commodity Price Shocks

In order to capture large adverse price movements, we define a negative price dummy along the lines of Collier and Goderis (2009). We first difference the commodity export price index to make it stationary, and then remove the predictable elements from the stationary process by running the following basic annual forecasting model:

$$\Delta TOT_{i,t} = \alpha_0 + \alpha_1 t + \beta_1 \Delta TOT_{i,t-1} + \beta_2 \Delta TOT_{i,t-1} + \varepsilon_{i,t} \qquad (2)$$

where $TOT_{i,t}$ is the log commodity export price index and t is a linear time trend. We collect the residuals $\varepsilon_{i,t}$ from (2) and derive the 5th and 95th percentile of their distribution. We next define positive and negative commodity export price shock episodes as the observations with residuals above the 95th percentile or below the 5th percentile, respectively. Having identified the shock episodes, we construct two variables. The first captures positive commodity export price shocks and equals the first log difference of the commodity export price index for the positive shock episodes, and 0 otherwise. The second captures negative commodity export price shocks and equals minus the first log difference of the commodity export price index for the negative shock episodes, and 0 otherwise.

In addition to actual shocks, we also include a measure of export price uncertainty. Following Dehn (2000), we use a GARCH (1, 1) model in which the actual volatility in a country's commodity export prices is explained by past volatility and past expected volatility:

$$\Delta TOT_{i,t} = \alpha_0 + \alpha_1 t + \beta_1 \Delta TOT_{i,t-1} + \beta_2 \Delta TOT_{i,t-1} + \varepsilon_{i,t} \quad (3)$$

$$\sigma_t^2 = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 + \gamma_2 \sigma_{t-1}^2 \quad (4)$$

Where σ_t^2 denotes the variance of $\varepsilon_{i,t}$, conditional upon information up to period *t-1*.

We use the fitted values of σ_t^2 in equation (4) as a measure of commodity export price uncertainty, because it captures the "predicted" variance of the innovations in commodity export prices from past actual and expected volatility. Intuitively, this makes use of the concept of volatility clustering: big shocks tend to be followed by big shocks in either direction. This implies that historical information about the volatility in commodity prices can be used to predict future volatility. We use the predicted future volatility as a measure of commodity price uncertainty.

In order to study the role of reserves as insurance against commodity price shocks we use an interaction term between negative price shock and a reserve dummy that takes a value of one if short term debt to GDP ratio is equal to one and zero otherwise. Our choice of the particular measure of reserve was guided by availability of reliable data for the relevant period as well as theory. Firstly, models studying the insurance value of reserves incorporate financing constraints such as limit on roll-over of short term debt as a key feature (see e.g. Jeanne and Ranciere, 2011). This is because the precautionary role of reserves becomes especially significant in situations of financing constraints of this sort. Second, literature has shown that short term debt and reserve accumulation can act as substitutes when it comes to exogenous shocks (see Alfaro and Kanczuk, 2009). Threshold value of one for short term debt to reserve ratio is one of the rules of thumb used by IMF and others to measure reserve adequacy.

2.2 Output Cost and Negative Price Shock

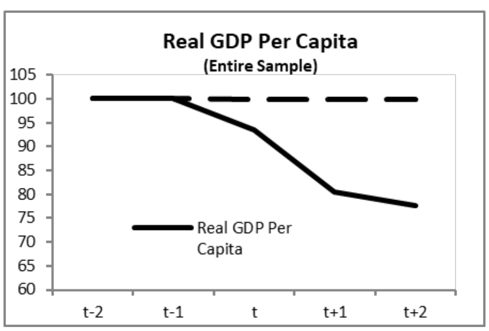
This section provides the preliminary empirical evidence supporting the threshold effects of reserve holdings discussed above. Figure [3] plots the path of real GDP per capita in the event of a large negative price shocks relative to the trend line for the countries under consideration. Time `t' is the point at which the country is hit by a negative price shock as defined in the previous section. Output per capita prior to the shock is normalized to 100 for all the countries in the sample. For each country, the trend path for real GDP per capita is obtained by using its average growth rate over the sample period. These individual paths are then averaged to obtain the dashed trend line. The solid line shows the actual path of average real GDP per capital around the shock for the countries under consideration.

The first plot shows the behavior of average Real GDP Per Capita along with the trend in the event of a negative price shock for all the countries in our sample. Real GDP per capita falls sharply below the trend line in the event of a negative price shock and remains below the trend line until two years after the shock. On an average, Real GDP per capita falls by 20 percent from its pre-shock level, a year after the shock. It falls by another 2 percent the next year. The cumulative loss in output over the three year period starting from the year of the adverse price shock is roughly 10 percent.

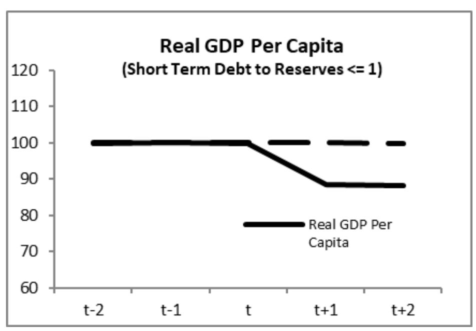
The second plot shows the same response function for countries with short term debt to reserve ratio below the prescribed Greenspan –Guidotti rule of one. For these countries, output falls by 11 percent a year after the shock and remains at that level for another year. Growth rate or real GDP per capita returns to its pre-shock level after one year. The cumulative loss in output is much less compared to the average (around 7 percent over the three years starting from the year of the shock). Finally, plot 3 shows the impulse response of real GDP per capita to a negative price shock for countries with short term debt to reserves ratio above 1. The cumulative loss in output during the first three years of the shock is roughly 30 percent in this case. Also, the growth rate of real GDP per capita remains below the trend level even two years after a negative price shock in case of these countries.

Figure [3]

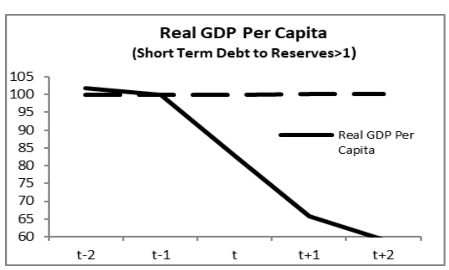
Impact of an adverse price shock on real GDP



(Entire Sample)



(Short Term Debt to Reserves <=1)



(Short Term Debt to Reserves >1)

Overall, large negative price shocks cause output to fall below the long term trend for commodity export dependent countries. Countries with short term debt to reserve ratio above one face a much higher loss in output due to large adverse price shocks. These threshold effects of reserves and debt levels need to be taken in to account while analyzing the adequacy of foreign reserves in developing countries.

2.3 Pooled Mean Group Estimation

We use the following empirical model to study the relationship between commodity price shocks and movements in real GDP per capita:

$$\Delta y_{i,t} = \alpha_i + \lambda y_{i,t-1} + \beta_1 x_{i,t} + \sum_{k=1}^{p} \gamma_k \Delta y_{i,t-k} + \sum_{k=0}^{q} \phi_k \Delta x_{i,t-k} + \beta_s S_{i.t} + u_{i.t} \quad (1)$$

for i = 1,…N and t = 1,…T , where $y_{i,t}$ is the log of real GDP per capita of country i in year t, α_i is a country-specific fixed effect, and $x_{i,t}$ is an $m \times 1$ vector of m variables that are expected to affect the GDP both in the short run and the long run. These include commodity export price index, deviations of world GDP per capita around long term trend as a measure of world demand and log of world oil price index. Finally, $S_{i.t}$ is an $n \times 1$ vector of n control variables that includes indicators of institutional quality, exchange rate regimes and dummies for large price shocks.

Table 2 presents the results for short term coefficients from this model. As we can see, the error correction term is negative and significant for both versions of the model indicating a long-run co-integrating relationship between the variables in question. Coefficients on current and lagged negative price shock terms are insignificant.

Our key variable of interest is the interaction term between the negative price shock and a dummy that takes a value one if short term debt to reserve ratio is above one and zero otherwise. The coefficient on this term and its lag is negative and for the lagged term the coefficient is significant. This indicates that countries with short term debt to reserves ratio above the Greenspan-Guidotti rule face a significantly higher loss of output in the event of a negative price shock. On an average, these countries face an output loss that about 2.5 percent points ($0.041 \times 0.60 = 0.0248$) higher than that for entire sample for an average sized commodity price shock.

Another significant finding of this exercise is the role of commodity price volatility. Higher commodity price volatility has a significant and negative impact on the growth of real GDP per capita apart from the level impact. This is in line with the findings in the existing literature. While we do not take this in to account for the modeling purpose, a fruitful future area of research could be to look in to the relationship between commodity price volatility and optimal reserves.

Our empirical results remain robust to the inclusion of other control variables such as trade openness and indicators of institutional quality. Inclusion of a fixed exchange rate dummy does however widen the output loss gap between the low reserve countries (those with short-term debt to reserves ratio >1) and the rest.

Table 2 gives the long run coefficients for the two models. Both commodity price and World GDP have positive coefficients indicating a positive long run relationship between them and the real GDP per capita. We do not find evidence for resource curse in our sample of countries.

2.4 Reserves and Current Account Adjustment

One of the ways in which lower short term debt to reserve ratio can help countries in the event of adverse price shock is through smoother adjustment of Current Account. Countries with higher level of reserves in relation to their external short term debt would, in principle, require smaller adjustment in their current account in the event of external shocks. There is evidence to this effect in the literature (Aizenman, 2008). We try to test this hypothesis using the data for 35 commodity exporting African countries and find strong evidence to the same effect. We use the following model to test our hypothesis:

$$\Delta CA_{i,t} = \alpha_i + \lambda CA_{t-1} + \beta_1 x_t + \sum_{k=1}^{p} \gamma_k \Delta CA_{i,t-k} + \sum_{k=0}^{q} \phi_k \Delta x_{i,t-k} + \beta_s S_{i.t} + u_{i.t}$$

for i = 1,…N and t = 1,…T , where $CA_{i,t}$ is the Current Account Deficit as a share of GDP of country i in year t, α_i is a country-specific fixed effect, and $x_{i,t}$ is an $m \times 1$ vector of independent variables that might affect the Current Account in the short run as well as the long run. These include the log of Short Term Debt to Reserve ratio plus one. Finally, $S_{i.t}$ is an $n \times 1$ vector of n control variables that include indicators of institutional quality, trade openness and an exchange rate regime dummy along with the price shocks.

The main focus of this exercise is to verify if the size of Current Account adjustment required in the event of a negative price shock depends on the short term debt to reserve ratio in the economy. One would expect countries with higher level of foreign reserves and lower level of short-term foreign debt to undergo a smaller adjustment in their current account in the event of adverse price shocks. In order to test this, we multiply the negative price shock dummy with the log of short term debt to reserve ratio plus one. A significant negative coefficient on this term implies that countries with higher short term debt to reserve ratio face a bigger adjustment in their current accounts in the event of negative price shocks. This can be one potential explanation for the difference in the output cost across countries with different levels of short term debt to reserve ratio.

Table [4] shows the result for GMM estimates of our model. As we can see, across the different specifications, the interaction term Negative price $shock_{t-1}$* $Ln(1+stdr_{t-1})$ has a significant negative coefficient implying that countries with a higher short term debt to reserves ratio face a much bigger adjustment in their current accounts in the event of an adverse price shock. This is in line with our expectations and can potentially explain the observed differences in output cost across countries with different levels of short-term debt to reserves ratio.

The next section presents a small open economy model to address the question of optimal level of reserves in light of the observed non-linear relationship between the level of reserves and output loss due to adverse price shocks.

3. Small Open Economy with Two Goods

Consider a Small Open Economy (S.O.E.) with two goods – one tradable and another non-tradable. The economy follows a deterministic path for the output of two goods, disturbed only by exogenous shocks to the terms of trade. Probability of the terms of trade shock is exogenously given as π_{TOT}. The domestic economy is composed of the private sector and the government.

3.1 Private Sector

The private sector is populated by a representative infinitely-lived consumer who consumes both tradable and non-tradable goods. The representative agent can engage in short-term borrowing from abroad to augment her consumption of the tradable good. This debt has to be repaid in terms of tradable good even when there is a shock to the terms of trade with an interest rate r. There is no default. In the event of a shock the agent repays his outstanding external debt but cannot engage in fresh borrowing during or after the shock. No-debt-roll-over assumption is necessary for keeping the reserve management problem meaningful since without it representative consumer will be able to smooth over his consumption by engaging in external borrowing. This assumption is less restrictive than it appears at first. As long as there is some limit on the ability of the economy to engage in fresh external borrowing at the time of shock our model can be used to study the problem of optimal reserves after minor changes in the benchmark calibration.

Consumption of the non-tradable good has to be equal to its production in any period. Denoting the periods before, during and after the shock with the sub-scripts *b*, *d* and *a* respectively, these assumptions can be expressed as follows,

$$L_{t,b} = \overline{L}, L_{t,d} = L_{t,a} = 0,$$

where L_t is the short term debt.

The representative private consumer is subject to the following budget constraints:

$$C_t^T = T \times Y_t^T + L_t - (1+r) \times L_{t-1} + Z_t \quad (1.1)$$

$$C_t^N = Y_t^N \quad (1.2)$$

Here C_t^T is the consumption of tradable good in period t and C_t^N is the consumption of non-tradable good. Y_t^T and Y_t^N are the period t output of tradable and non-tradable goods respectively. T is the terms of trade and Z_t is the transfer of tradable good by the government in period t. In every period the consumption of non-tradable good is equal to its output (for simplicity we assume that the non-tradable good cannot be saved). The consumption of tradable good on the other hand equals the sum of the output of tradable good, net new external borrowing and government transfer of tradable good.

Combining equations 1.1 and 1.2 we get the overall budget constraint for the consumer,

$$C_t^T + P_t^N \times C_t^N = T \times Y_t^T + L_t - (1+r) \times L_{t-1} + Z_t + P_t^N \times Y_t^N \quad (1.3)$$

P_t^N is the endogenously determined price of non-tradable good in terms of tradable good.

Output of tradable as well as non-tradable good grows at the same constant rate `g' until terms of trade shock occurs. The terms of trade shock is associated with a fall in output growth by γ. After the shock, output growth returns to its long-run path. Loss in output growth due to the terms of trade shock depends on the level of reserves as measured by short-term debt to reserves ratio. As shown above , countries with short-term debt to reserve ratio above one face a significantly higher loss in output due to exogenous price shocks compared to countries with a lower short term debt to reserves ratio.

Once again, denoting the periods before, during and after the shock with the sub-scripts b, d and a; we can write the following equations summarizing our assumptions,

$$Y_{t,b}^T = (1+g)^t Y_0^T, Y_{t,d}^T = (1-\gamma)(1+g)^t Y_0^T, Y_{t,a}^T = (1+g) Y_{t-1,d}^T \quad (1.4)$$

$$Y_{t,b}^N = (1+g)^t Y_0^N, Y_{t,d}^N = (1-\gamma)(1+g)^t Y_0^N, Y_{t,a}^N = (1+g) Y_{t-1,d}^N \quad (1.5)$$

If short term debt to reserves ratio > 1 $\gamma = \gamma_1$,

Otherwise $\gamma = \gamma_2$

Finally, $\gamma_1 > \gamma_2$

3.2 Government

Role of the government in our model is that of an insurance provider. It does so by issuing a long-term security that does not have to be repaid during the shock. The long-term security issued by the government is a bond that yields one unit of good in every period until the shock occurs. The security stops yielding any income after the shock.

The pre-shock price of the security is equal to the present discounted value of the one unit of good it pays in the next period plus the expected market value of the security,

$$P = \frac{1}{1+r+\delta}\left[1+(1-\pi_s).P\right] \qquad (1.6)$$

Where $\pi_s = \pi_{TOT}$ is the probability of a `terms of trade' shock r is the short term interest rate (equal to the discount rate of the representative consumer) and δ is the insurance premium. This implies,

$$P = \frac{1}{r+\delta+\pi_s} \qquad (1.7)$$

Equation 1.7 uses the fact that the price of the long-term security is constant before the shock and falls to zero when shock occurs.

The government issues the long-term security to finance a stock of reserves,

$$R_t = PN_t \qquad (1.8)$$

Where N_t is the number of securities issued by the government in period t. Government's budget constraint is given by:

$$Z_t + R_t + N_{t-1} = P(N_t - N_{t-1}) + (1+r)R_{t-1} \qquad (1.9)$$

Equation 1.8 can be used to substitute out N_t and N_{t-1} from equation 1.9 to get the following expression

$$Z_t^b = -\left(\frac{1}{P} - r\right)R_{t-1} = -(\delta + \pi_s)R_{t-1} \qquad (1.10)$$

Negative transfer implies that the government taxes the representative consumer in order to pay for the cost of carrying the reserves, which is proportional to the insurance premium plus the probability of a shock.

If and when the shock occurs, the government transfers the reserves (net of last payment on the long-term security) to help the representative consumer,

$$Z_t^d = (1-\delta-\pi_s)R_{t-1} \tag{1.11}$$

After the shock the governments become inactive R_t, N_t and Z_t are all equal to zero.

Using equations 1.10 and 1.11 we can get the expressions for tradable consumption before and after the shock,

$$C_{t,b}^T = T \times Y_{t,b}^T - (\delta+\pi_s)R_{t-1} \tag{1.12}$$

$$C_{t,d}^T = (1-\gamma_{TOT})T \times Y_{t,d}^T + (1-\delta-\pi_s)R_{t-1} \tag{1.13}$$

$$C_{t,a}^T = T \times Y_{t,a}^T \tag{1.14}$$

Increasing R_{t-1} raises the consumption of tradable good in period t if there is a shock and lowers it if there is no shock. Expression for the consumption of non-tradable is as follows,

$$C_{t,b}^N = Y_{t,b}^N,\ C_{t,d}^N = Y_{t,d}^N,\ C_{t,a}^N = Y_{t,a}^N \tag{1.15}$$

Government chooses R so as to maximize the welfare of the representative consumer,

$$U_t = \sum_{s=0}^{\infty} E_0\left[(1+r)^{-s} u(C_{t+s})\right] \tag{1.16}$$

Where the flow utility function has a constant relative risk aversion σ,

$$u(C) = \frac{C^{1-\sigma}-1}{1-\sigma} \tag{1.17}$$

And C is the aggregate consumption,

$$C_{t,d} = (C_{t,d}^T)^{\alpha} \times (C_{t,d}^N)^{1-\alpha} \tag{1.18}$$

$C_{t,d}$ is the commonly used Cobb-Douglas consumption aggregator with constant elasticity of substitution between the tradable and the non-tradable goods and α is the share of tradable good in total consumption. The discount rate of representative consumer is equal to the rate of interest r. This ensures that the consumer's maximization problem is well defined in the absence of endogenous discount rate or upward sloping interest rate function. Since R_t only affects the level of consumption in the next period, government's problem is to choose the level of R_t in each period that maximizes the level of expected utility $u(C_{t+1})$ next period.

$$R_t = \arg\max\left(E_0\left[u(C_{t+1})\right]\right) = \arg\max(1-\pi_s)\times u(C_{t+1.b}) + \pi_s \times u(C_{t+1,d}) \tag{1.19}$$

The first order condition for the problem is,

$$(1-\pi_s)(\delta+\pi_s)\times u_T^{'}(C_{t+1,b}) = (1-\delta-\pi_s)\times \pi_s \times u_T^{'}(C_{t+1,d}) \tag{1.20}$$

The right-hand side of equation 1.20 is the expected marginal utility of reserves conditional on a shock. The left-hand side is the probability of no shock times the expected marginal cost of reserves conditional on there being no shock. When the level of reserves is chosen optimally, the marginal utility of holding reserves is equal to the marginal cost of holding them.

Denoting the marginal rate of substitution between consumption in the event of a shock and consumption in the absence of a shock by p_t we can write,

$$p_t = \frac{E_0\left[u_T^{'}(C_{t,d})\right]}{E_0\left[u_T^{'}(C_{t,b})\right]} \tag{1.21}$$

Where `*d*' denotes `during the shock' and `*b*' denotes `before the shock'. Equation 1.20 says that when reserves are set optimally, this price should be constant and equal to,

$$p = \frac{\delta+\pi_s}{1-\delta-\pi_s} = \frac{1}{1-\delta-\pi_s} - 1 \tag{1.22}$$

Some Notations,

$$\begin{aligned} C_{t,d} &= \left(C_{t,d}^T\right)^{\alpha}\left(C_{t,d}^N\right)^{1-\alpha} \\ C_{t,b} &= \left(C_{t,b}^T\right)^{\alpha}\times\left(C_{t,b}^N\right)^{1-\alpha} \end{aligned} \tag{1.23}$$

$$E_0\left[u_T^{'}(C_{t,d})\right] = \alpha\times\pi_s\times\left(\left(C_{t,d}\right)^{-\sigma}\times\left(C_{t,d}^T\right)^{\alpha-1}\times\left(C_{t,d}^N\right)^{1-\alpha}\right) \tag{1.24}$$

And,

$$E_0\left[u_T^{'}(C_{t,b})\right] = \alpha\times(1-\pi_s)\times\left(\left(C_{t,b}\right)^{-\sigma}\left(C_{t,b}^T\right)^{\alpha-1}\left(C_{t,b}^N\right)^{1-\alpha}\right) \tag{1.25}$$

Using equations 1.21, 1.23, 1.24 and 1.25, we can simulate the optimal level of reserves for different values of parameters and shocks. Section 3 discusses the benchmark calibration and the simulation results. As expected, results are sensitive to the choice of parameter values. Hence, we also present results for country specific parameter values.

3.3 Extensions

3.3.1 Real Exchange Rate Depreciation

We consider valuation effects caused by real exchange rate depreciation at the time of the commodity price shock. Let us assume that the country's external liabilities and reserves are denominated in foreign currency. Then the budget constraint of the representative consumer (1.1) is replaced by,

$$C_t^T = T \times Y_t^T + RER_t\left(L_t - (1+r) \times L_{t-1} + Z_t\right) \qquad (1.26)$$

Real exchange rate is constant in normal times (equal to 1) and depreciates by ΔRER_t at the time of the terms of trade shock, i.e., $RER_t^b = 1, RER_t^d = 1 + \Delta RER_t$.

Impact of real depreciation on the optimal level of reserves will depend upon its impact on the size of required BOP adjustment on one hand and its impact on the value of the reserves in terms of tradable consumption on the other. Real exchange depreciation raises the required BOP adjustment since external debt is denominated in foreign currency. At the same time, the level of insurance provided by the reserves also goes up as a result of real exchange depreciation since reserves are denominated in foreign currency too. The sign and size of the net effect are explored below.

3.3.2 Endogenous Risk Premium

So far we had assumed that the cost of holding reserves as insurance against price shock, δ, is exogenous. However, as shown by Jeanne and Ranciere (2011), the commonly used measure of this cost – the difference between the interest on external debt and return on reserve assets – does not capture the true 'risk premium'. `True risk premium' is given by the difference between the actual spread and the actuarially fair price for insurance. Assuming that the risk premium would be positively correlated to the level of risk, it is no longer necessary that the countries with greater risk should carry more reserves. We now endogenize the pure risk premium so that it becomes a function of behavioral parameters and the probability of shock.

To do our analysis we use the method adopted by Jeanne and Ranciere (2011) who assume overlapping generations of insurance providers providing insurance to price taking insurance buyers. Under certain assumptions one can replicate the outcome of such an insurance contract by assuming that the consumers can insure themselves against price shocks by issuing a contingent debt contract whose principle is repaid only if there is no price shock and investing the proceeds in riskless 'reserve assets' that provide a rate of return r. The insurance provider tries to maximize his utility from the insurance contract by equating the marginal utility of return from the insurance contract and the riskless asset – the only two types of assets available.

Supply of insurance under these assumptions is given by:

$$\rho_t = \varpi_t \frac{1 - p_t^{1/\sigma^*}}{1 - \left(1 - p_t^{1/\sigma^*}\right) x_t} \qquad (1.27)$$

Here, ϖ_t is the ratio of foreign insurer's total wealth to domestic output, σ^* is the coefficient of risk aversion for the foreign insurer; x_t is the spread between the riskless interest rate r and the contingent debt contract, and p_t is the price of reserves. Equilibrium price and quantity of insurance are obtained by equating the supply and demand for insurance (See Appendix for details of the derivation). We then do sensitivity analysis of our benchmark results.

4. Simulation Results

The two-good model presented in the above paragraphs does not allow for an analytical solution. We therefore use numerical techniques to solve for the level of optimal reserves as a function of output.

4.1 Choice of Parameters

Table A.1 gives the value of key parameters used for benchmark simulations. These parameter values have been calibrated using data for 35 SSA countries. Average size of the terms of trade 'shock' across African countries between 1970 and 2007 was about 60 percent (4-5 percent of G.D.P). This was used to calibrate γ_{TOT}. Similarly average probability of terms of trade shock was used to calibrate π_{TOT}, which turned out to be around 5 percent. Potential output growth, g, was set to be 5 percent based on the average growth rate for Africa over last 5 years. Ratio of short term debt in GDP, λ, was again calibrated using data for the 35 SSA countries over 2004-2010 which was equal to 0.05. Risk free short-term rate of return, r, is set at 5 percent – same as the federal funds rate during 1987-2005. Term premium, δ, which reflects the cost of holding reserves, is set at 3 percent. This is twice the value used by Jeanne and Ranciere (2011) and reflects the fact that the 'opportunity cost' of holding foreign reserves is higher in low-income African countries than in emerging market countries. Output cost of terms of trade shocks, γ_1 and γ_2 were calibrated based on the PMG estimates presented above. γ_1 is 2. 5 percent and γ_2 is about 0.36 percent. The remaining parameters are obtained from other low-income country studies. We check the sensitivity of our results to some of these parameter choices.

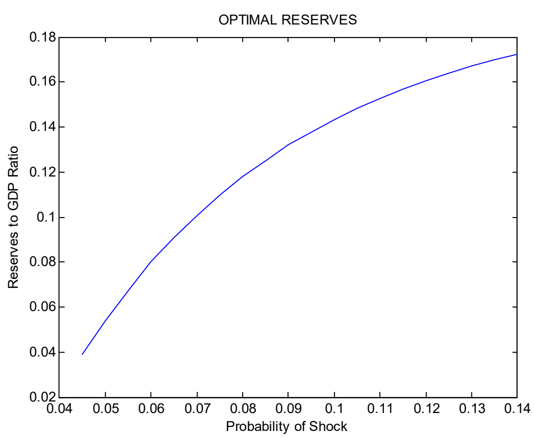
4.2 Benchmark Results and Sensitivity Analysis

Based on our benchmark calibration, the optimal level of reserves for Sub-Saharan African countries is 16.5% of GDP. This is close to the average reserve to GDP ratio for Sub-Saharan Africa over last five years (14.5 %). Thus, our model can explain the observed reserve levels in sub-Saharan African countries during recent years as an outcome of a desire to insure against commodity price shocks. Actual reserves to GDP ratio for Sub-Saharan Africa is in fact slightly lower than the value predicted by the benchmark model indicating that unlike the situation in emerging market countries, recent reserve accumulation in Africa is not excessive. Compared to the estimates of optimal reserves for emerging market countries provided by Jeanne and Ranciere (2011), the optimal level of reserves for African commodity exporters is slightly higher.

Of course, the optimal level of reserves would vary depending upon the size and the probability of terms of trade shocks, cost of holding reserves and output cost. Figure 4 plots the optimal level of reserves to GDP as a function of key parameters of the model.

Figure [4]

Benchmark Simulations



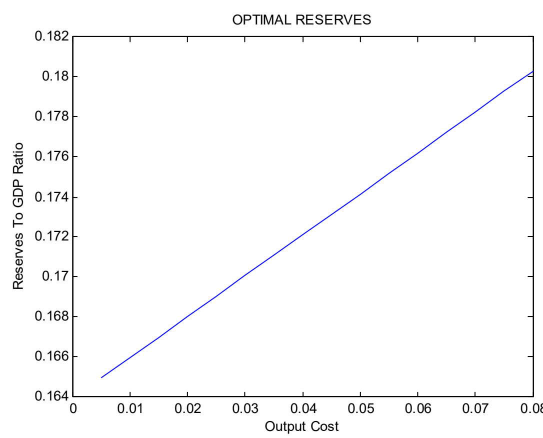


Figure [4]

Benchmark Simulations (Contd.)

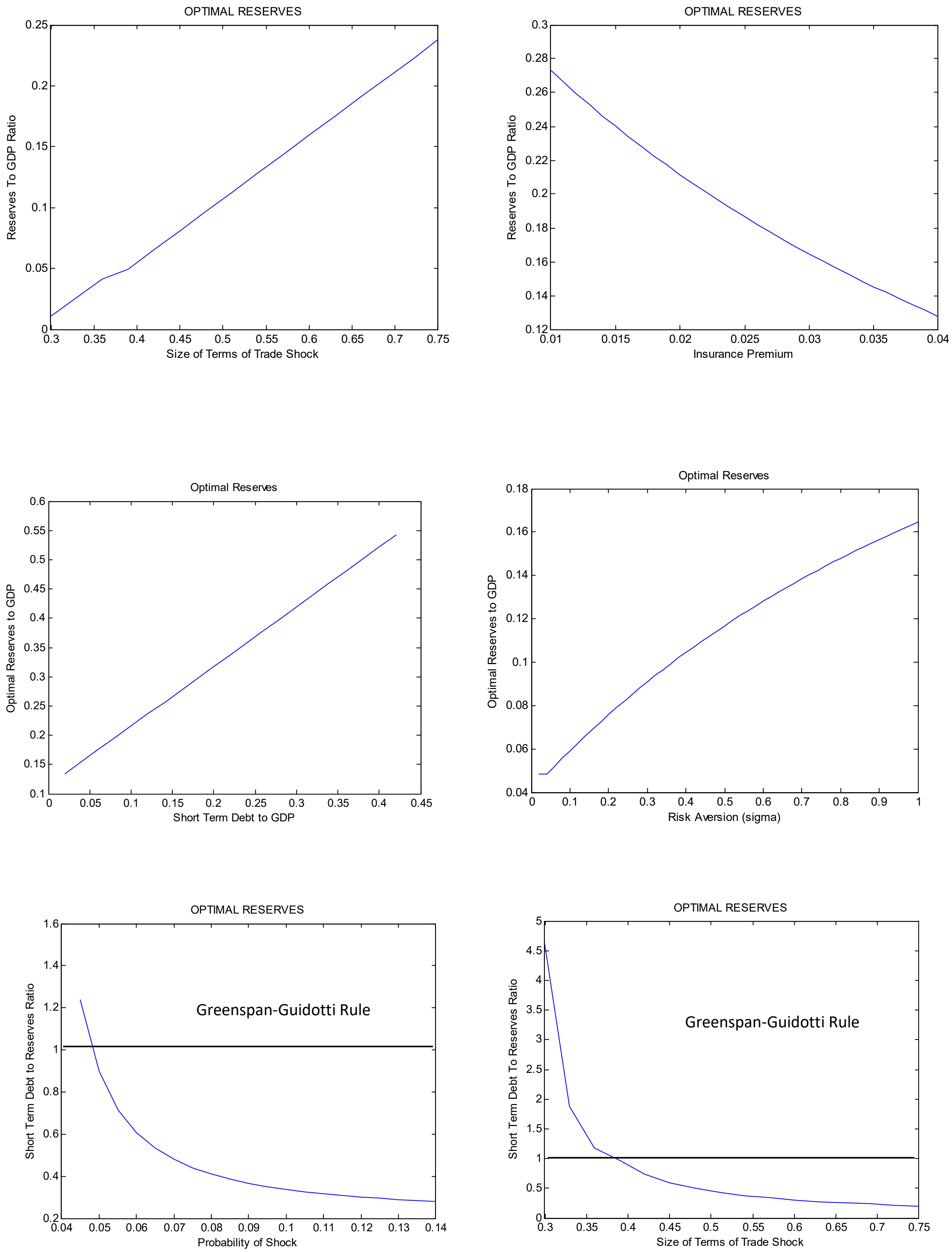
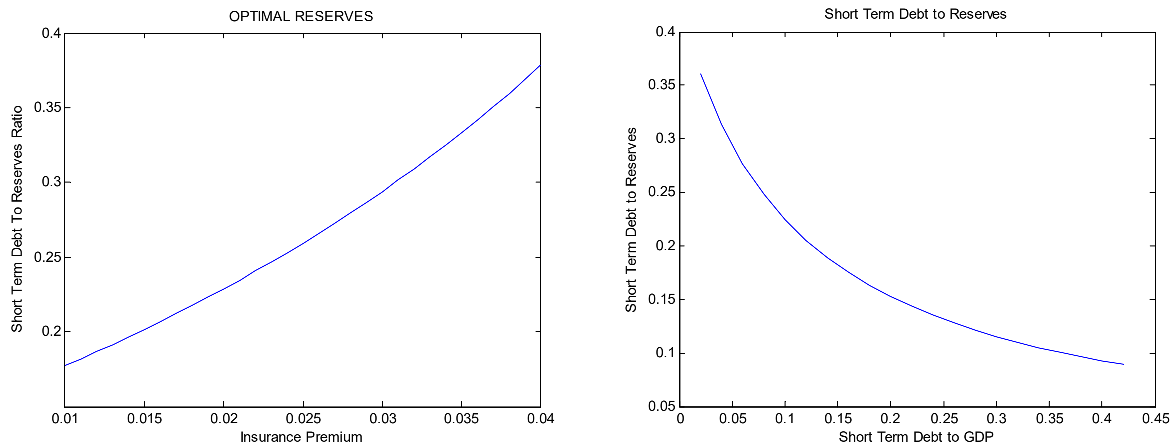


Figure [4]

Benchmark Simulations (Contd.)



As we can see higher the probability of terms of trade cost and larger the size of terms of trade shock, higher the optimal level of reserves. Higher cost of reserve holding (δ) implies lower level of optimal reserves while higher output cost of adverse price shock implies higher level of optimal reserves.

Similarly, higher short-term debt to GDP ratio implies a higher level of optimal reserves. Thus, developing countries face a trade-off between higher level of external debt and greater amount of foreign reserves needed as insurance. Finally, higher degree of risk aversion implies higher level of optimal reserves.

We can see that the optimal short-term debt to reserves ratio is equal to the Greenspan-Guidotti rule of one only for particular values of parameters. Hence it is important to take in to account country specific characteristics in order to determine the optimal level of reserves. Next, we present the extensions to our benchmark model.

Figure 5 plots the optimal level of reserves as a function of exchange rate depreciation. For our benchmark parameter values we find that the optimal level of reserves falls slightly with the extent of real exchange rate depreciation. In other words, greater the extent of real exchange rate depreciation accompanying an adverse price shock, smaller is the optimal level of reserve holdings. Real exchange rate depreciation helps commodity exporters by increasing the insurance value of their reserve holdings. However, the benefit of real depreciation in terms of smaller required reserves decreases as the size of real depreciation increases. In other words, for large depreciations, the valuation impact on reserves is much smaller compared to the required BOP adjustment. To the extent that a more flexible exchange rate regime allows for greater adjustment in real exchange rate, it reduces the level of foreign exchange reserves required as insurance against price shocks.

Figure [5]

Real Exchange Rate Depreciation

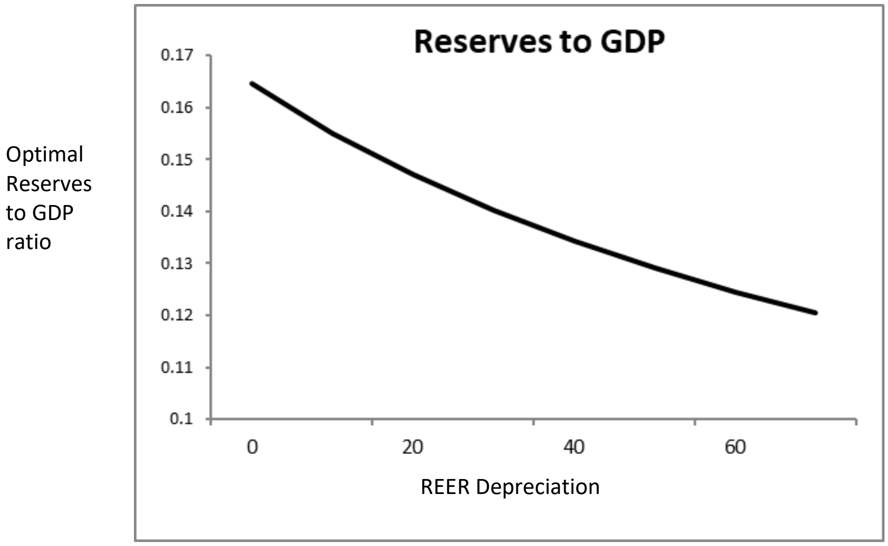
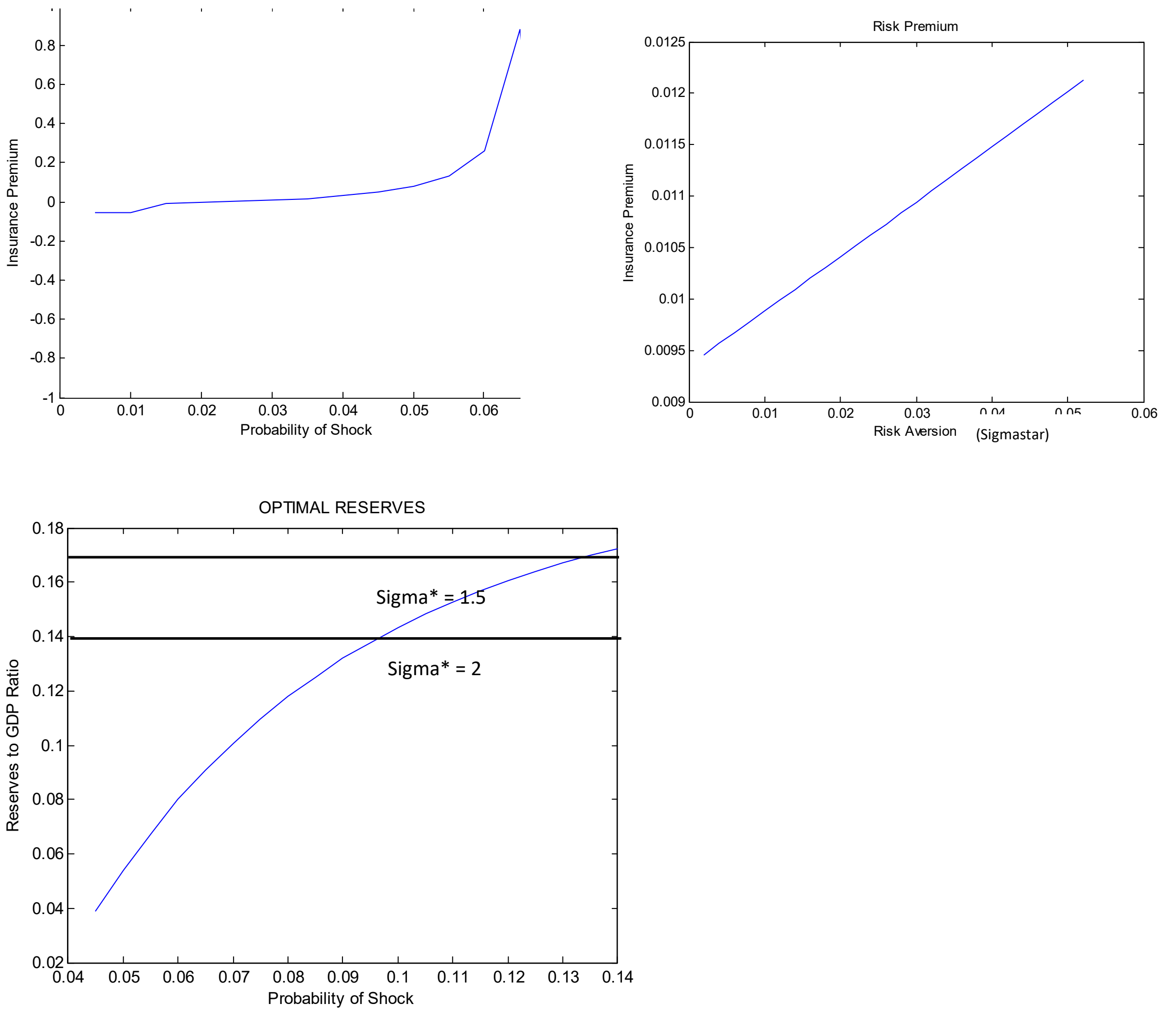


Figure 6 presents the results for the case when the `risk premium' is endogenous. The first two panels of the figure show the variation in endogenously determined risk premium with a change in the probability of price shock and the degree of foreign insurer's risk aversion. As expected, 'pure' risk premium increases with the probability of price shock and the degree of foreign insurer's risk aversion. Relationship between the probability of shock and risk premium is not linear though.

The last panel shows the variation in the optimal level of reserves with a change in σ^*. Higher degree of risk aversion amongst foreign insurers implies a larger 'risk premium' which implies a lower level of `optimal reserves' (σ^* = 2, Optimal reserves to GDP = 13.9%; σ^* = 1.5, Optimal reserves to GDP = 16.7%). Thus, in situations where the foreign insurers become more risk averse due to greater uncertainty or some other exogenous reasons, the risk premium goes up while the optimal level of reserves goes down.

Figure [6]

Endogenous Risk Premium



One can infer the risk aversion parameter for foreign insurers by looking at the observed reserve levels. In our case a risk aversion parameter (σ^*) of 2 can help explain the observed level of reserve holding in Sub-Saharan Africa.

4.3 Country Specific Applications

Above results show that choice of key parameters can affect the level of reserves that would be optimal for a particular country. In this section we use some country specific parameters alongside a few common parameters to obtain country specific estimates of optimal reserves for the SSA countries. We use data from 1980-2011 to estimate the probability of terms of trade and aid `shocks' (where shocks are as defined in the beginning) and the share of tradable goods in consumption ($\alpha$) for each of the 42 SSA countries. Probability of a `shock' is simply the number of shock events during 1980-2011 divided by the total number of years for each country whereas the share of tradable consumption is calculated by multiplying the share of consumption in domestic demand with the share of imports in total consumption. Term premium δ is set equal to 0.3 for all countries except South Africa for which it set equal to 1.5 (same as that used for emerging markets by Jeanne and Ranciere)[2]. Figure 7 plots the level of actual reserves at the end of year 2011 against the optimal reserve level, as determined by our model, for SSA countries. Both, the actual and the optimal level of reserves are expressed as a ratio of GDP.

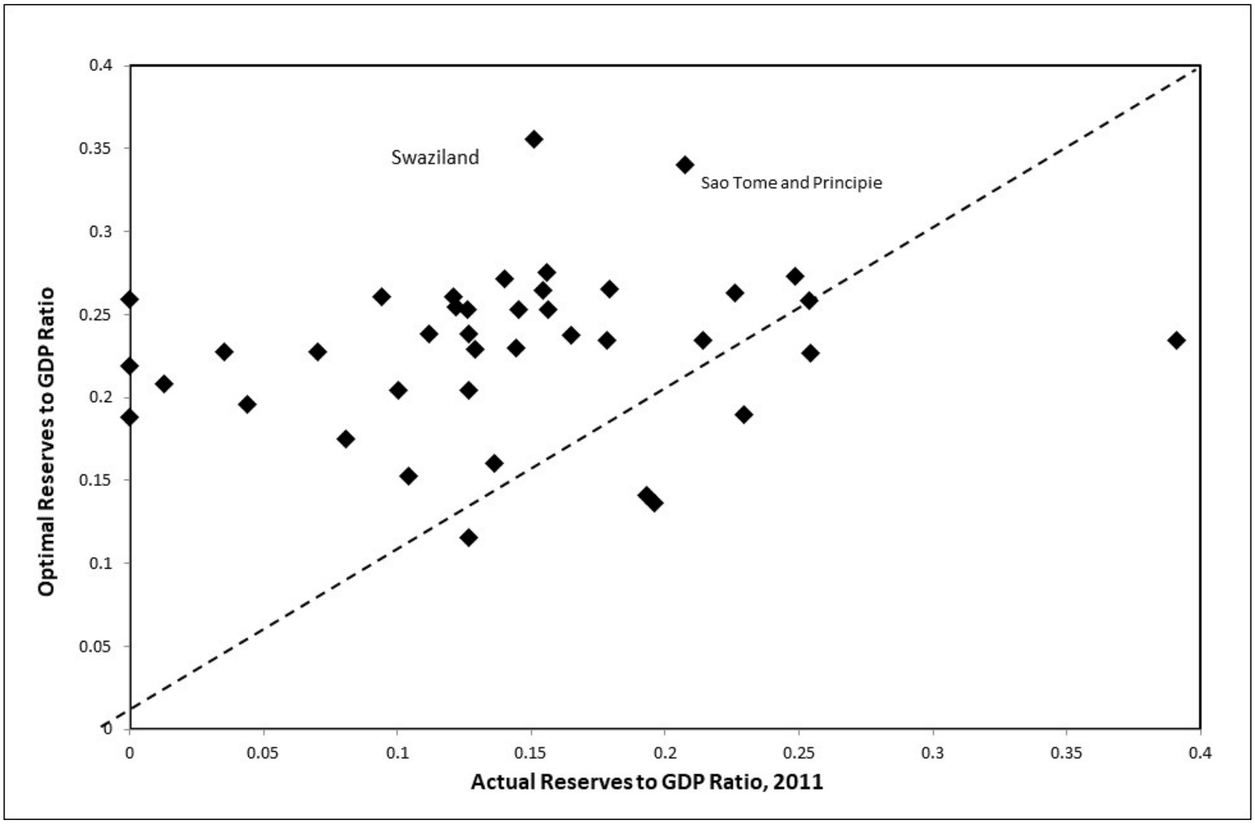
The broken line is the 45 degree line which identifies countries holding actual reserves equal to the optimal level for them. Countries to the right of this blue line hold more reserves than the optimal level indicated by our model. This can be due to several reasons – an un-anticipated surge in price of oil or other major exports increasing the government revenues or domestic money supply (thereby forcing government to undertake sterilization operations) or excessive dependence on a non-renewable resource for export revenues (e.g. diamonds in Botswana) that is likely to be exhausted in foreseeable future.

Countries to the left of the forty-five degree line are those carrying fewer reserves than suggested by our model. A few of these, such as Swaziland and Sao Tome and Principe seem to have reserve levels that are way below the optimal. This implies that these countries will not be in a position to face a large T.O.T shock, if and when it occurs. Possible reasons for such low level of reserves can be structural constraints on the capacity of the government to raise taxes and generate revenues or uninformed economic policies. Deciding upon the actual reasons and remedies for inadequate reserve accumulation requires us to look at each country separately.

[2] List of country specific parameters is available upon request from the authors.

Figure [7]

Reserve Adequacy for African Countries Using Country Specific Parameters /13



Source: W.E.O and World Bank database

Figure 8 illustrates the application of our model for assessing reserve behavior for individual countries with the example of a set of four countries. It plots 'reserve gap' for Angola; Congo, D.R.; Malawi and South-Africa for years 2000-11.

Reserve gap is defined as the difference between optimal level of reserves, as suggested by our model and the actual level of reserves. We use a combination of country specific and common parameters to simulate optimal level of reserves for each country over time. In particular, probability of terms of trade shocks, ratio of short term debt to G.D.P. and share of imports in consumption are country specific while the other parameters are common across countries. Out of country specific parameters, share of short term debt in G.D.P. is allowed to vary across time while other parameters remain constant.

[3] Similar picture emerges when we look at the short term debt to reserves ratio.

Having thus calculated the level of optimal reserve as a share of output for each country we subtract the actual level of reserves, also expressed as a share of output, to obtain the reserve gap. The reserve gap has declined for all countries in question except Malawi during the period under consideration. The decline in the reserve gap was most significant for Angola and Congo, Democratic Republic, where the optimal level of reserves declined in line with falling short term debt over this period. As a result Angola managed to reduce its `reserve gap` from twenty-seven percent to 0.5 percent of GDP while Congo, D.R. reduced the reserve gap from twenty six percent to nine percent of GDP.

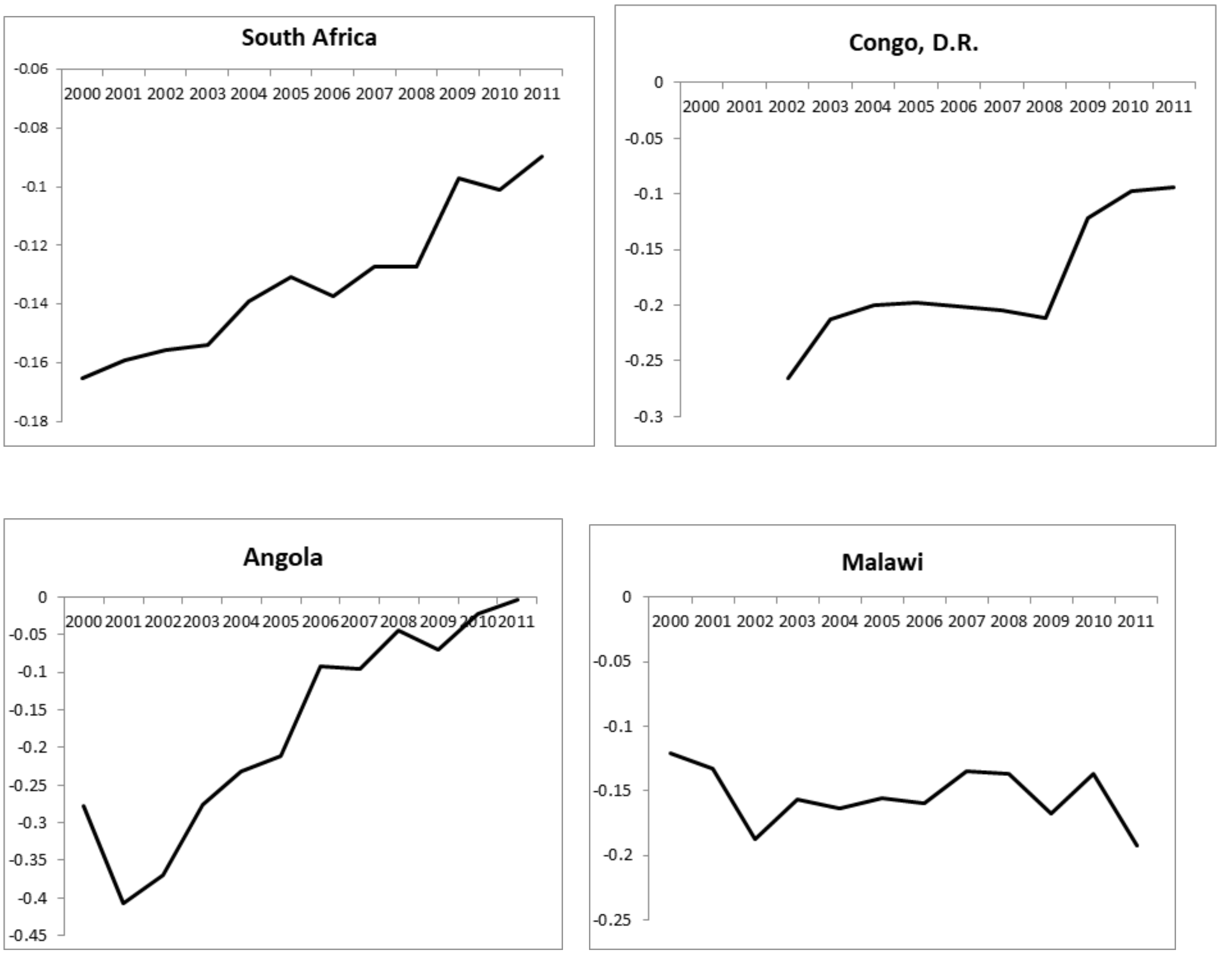
The level of foreign reserves held by South Africa increased from about 5 percent of GDP in 2000 to more than 10 percent of GDP in 2011. Over the same period the optimal level of reserves for South Africa declined from twenty one percent to nineteen percent due to a reduction in short term external debt. As a result, the reserve gap for South Africa declined from more than sixteen percent in 2000 to around nine percent of GDP in 2011.

The reserve gap for Malawi increased from twelve percent in 2000 to more than nineteen percent in 2011as the actual reserves to GDP ratio declined from thirteen percent to roughly 3.5 percent over the same period. Of the four countries in our sample Malawi is only one that has seen deterioration in its reserve position over the past decade. Considering the significant costs of carrying inadequate reserves in the event of an adverse price shock this definitely should be a cause of concern for the policy makers.

To summarize, assessing the adequacy of reserves held by a country requires us to look in to country specific factors affecting the reserve accumulation behavior. The two-sector model, with exogenous terms of trade and aid shocks is a good starting point in this direction which can be built upon by using more country specific information on factors affecting reserve accumulation.

Figure [8]

Country Specific Application – Illustrative Examples



Source: W.D.I and Author's Calculation (February, 2013)

5. Conclusion

This paper presents a small open economy model that can be used to derive the optimal level of reserves for primary commodity exporting countries subject to large commodity price shocks. The model assumes that reserves provide insurance; allowing countries to smooth domestic absorption against the disruption induced by a large adverse price shock associated with a fall in output.

We find that for plausible values of model parameters, the model can account for the average level of reserve holdings by Sub-Saharan African countries during recent years. Simulation for individual countries using country specific parameters allows us to capture the differences in their reserve adequacy levels. There are certainly various reasons for reserve accumulation other than insuring against commodity price shocks and in order to explain reserve accumulation by individual countries one would have to take in to account their unique macroeconomic environment including nature of their exchange rate regime, degree of openness and financial market integration besides other things. Our model, however, does provide a benchmark for policy makers trying to ascertain the adequacy of reserve levels in low-income commodity exporters in Africa. It would also be interesting to extend this model to take in to account large commodity price booms.

References

Joshua, Aizenman, 2008, *International Reserve Management and the Current Account*, Central Banking, Analysis, and Economic Policies Book Series, in: Kevin Cowan & Sebastián Edwards & Rodrigo O. Valdés & Norman Loayza (Series Editor) & Klaus Schmid (ed.), Current Account and External Financing, edition 1, volume 12, chapter 11, pages 435-474 Central Bank of Chile

Aizenman, Joshua and Riera-Crichton, 2008, `Real Exchange Rate and International Reserves in an Era of Growing Financial and Trade Integration', *The Review of Economics and Statistics*, Volume 90, Issue 4, Pages 812-815

Alfaro, Laura and Kanczuk Fabio, 2009, `Optimal Reserve Management and Sovereign Debt', *Journal of International Economics*, Volume 77, Issue 1, February 2009, Pages 23‑36

Caballero, Ricardo J. and Stavros Panageas, 2007, "A Global Equilibrium Model of Sudden Stops and External Liquidity Management," manuscript, MIT, Dept of Economics

Collier, Paul and Goderis, Benedikt, 2009, `Does Aid Mitigate External Shocks?', *Review of Development Economics,* Volume 13, Issue 3, August 2009, Pages 373–542

Deaton, Angus and Miller, Ron, 1995, "International commodity Prices, Macroeconomic Performance and Politics in Sub-Saharan Africa", *International Finance Section*, Department of Economics, Princeton University

Dehn, Jan, 2000, `The Effects on Growth of Commodity Price Uncertainty and Shocks,' World Bank Policy Research Working Paper No. 2455

Durdu, Ceyhun Bora, Mendoza, Enrique and Marco Terrones, 2007, "Precautionary Demand for Foreign Assets in Sudden Stop Economies: an Assessment of the New Mercantilism," *NBER Working Paper* 13123, National Bureau of Economic Research, Inc.

Pesaran, M. and Smith, Ron, 1995,`Estimating long-run relationships from dynamic heterogeneous panels,' *Journal of Econometrics,* Volume 68, Issue 1, July 1995, Pages 79‑113

Raddatz, Claudio, 2007, `Are External Shocks Responsible for the Instability of Output in Low-Income Countries?', *Journal of Development Economics*, Volume 84, Pages 155-187

APPENDIX I

Commodity Price Shocks

Aggregate Terms of Trade indices are unsuitable as proxies for commodity price movements as they include changes in non-commodity export prices. To address this issue we construct commodity export price indices for each country using commodity price data for 49 commodities. In particular we use the index first suggested by Deaton and Miller (1995). The index is structured as follows:

$$DM = \prod_i P_i^{W_i}$$

W_i is a weighing item and P_i is the dollar international commodity price for the commodity i . Dollar prices measure *c.i.f.* border prices and the weighing item W_i .is the value of commodity i in the total value of all commodities, n, for the constant base period j.

$$W_i = \frac{P_{ji} Q_{ji}}{\sum_n P_{jn} Q_{jn}}$$

In our case we use 1990 as the base year. In case of countries for which export data for the year 1990 was not available, we chose the closest year available to 1990. Since W_i is unique for each country, each country has its own commodity price index. Using world commodity prices has the advantage that these prices are not affected by the circumstances in individual countries (except where the exporting country enjoys some amount of market power) and therefore changes in these prices are truly exogenous. Use of constant base year weights does not capture shifts in the structure of trade. However, to the extent that we want to focus on the effects of exogenous price changes alone, this may actually be a good thing. Finally we deflate the commodity export price index for each country with the Unit value index of manufactured goods exports by developed economies (MUVI hereafter) to get the real commodity price index.

Technical Appendix: Endogenous Risk Premium

As shown by Jeanne and Ranciere (2011), the reserve insurance contract in the benchmark model can be replicated if the representative consumer can issue debt whose principal is repaid only if there is no sudden stop. One unit of this debt issued in period t has a face value of 1, and yields - $1 + r + x_t$ if there is no sudden stop in period $t + 1$, and $r + x_t$ if there is a sudden stop. Variable x_t, thus, is the interest rate spread on the consumer's external debt. Consumer sells this debt to foreign investors at the unitary price of 1 and invests the proceeds in reserve assets that yield a riskless return of r. The net payoff for the consumer in period $t + 1$ is

$(1+r)R_t - (1+r+x_t) R_t = - x_t R_t$ if no sudden stop (a.1)

$(1+r)R_t - (r+x_t) R_t = (1- x_t)R_t$ if sudden stop (a.2)

The insurance is provided by overlapping generations of foreign insurers. The generation born at t consumes in period $t + 1$. Each insurer born at t is endowed with w_{t+1} in period $t + 1$ and consumes $w_{t+1} + z_{t+1}$, where z_{t+1} is the transfer from the insurance contract with the representative consumer, given by

$z_{t+1} = -x_t r_t$ if no sudden stop,

$z_{t+1} = (1-x_t r_t)$ if sudden stop

Here r_t is the supply of reserves per insurer. The supply of insurance per insurer is a solution to the problem,

$$\max_{r_t} (1-\pi_t)\upsilon_t(w_{t+1} + x_t r_t) + \pi_t \upsilon_t(w_{t+1} - (1-x_t) r_t) \qquad \text{(a.3)}$$

Where υ_t is the utility function of an insurer born at t. If υ_t is CRRA with risk aversion σ^*, the first order condition is,

$$(w_{t+1} + x_t r_t)^{-\sigma^*} = p_t (w_{t+1} - (1-x_t) r_t)^{-\sigma^*} \qquad \text{(a.4)}$$

Solving a.4 for r_t gives the insurance supply curve 1.27.

Using the fact that $\delta_t = x_t - \pi_t$, we can express 1.27 as a function of δ_t and π_t. Equating the demand and supply of insurance gives us the equilibrium level of insurance and risk premium.

Table [2]

Pooled Mean Group Estimation

Pooled Mean Group Estimation Short Run Coefficients		
Variable	Model 1	Model 2
EC	-0.21*** (0.00)	-0.18*** (0.00)
△CTOT	-0.03 (0.10)	-0.03 (0.11)
△WGDPpc	0.15 (0.47)	0.27 (0.25)
Negative price shock	-0.012 (0.41)	-0.015 (0.31)
Negative price shock $_{t-1}$	0.013 (0.35)	0.009 (0.47)
Negative price shock*Reserve Dummy	-0.017 (0.18)	-0.013 (0.61)
(Negative price shock*Reserve Dummy)$_{t-1}$	-0.025** (0.034)	-0.022* (0.06)
Volatility of TOT	-1.07*** (0.00)	-1.17*** (0.00)
Fixed Exchange Rate	0.01** (0.05)	
Log Likelihood	2231	2110

Table [3]

Long Run Coefficients

Long Run Coefficients		
Variable	**Model 1**	**Model 2**
Y	1	1
TOT	0.10*** (0.00)	0.11*** (0.00)
WGDPPC	1.52*** (0.00)	1.3*** (0.00)

Table [4]

GMM Estimates

Dependent Variable: Change in Current Account Deficit

EC_{t-1}	-0.00*** (0.00)	-0.00*** (0.00)	-0.00*** (0.00)
Δ Capital Account $_{t-1}$	-0.25*** (0.00)	-0.25*** (0.00)	-0.24*** (0.00)
Δ $Ln(1+stdr_{t-1})$[4]	-1.84*** (0.001)	-1.84*** (0.001)	-2.25*** (0.001)
Negative Price Shock $_t$	-7.7 (0.12)	-7.7 (0.12)	-7.5 (0.14)
Negative Price Shock $_{t-1}$	4.9 (0.24)	4.9 (0.24)	5.2 (0.23)
Negative Price Shock $_t$ * $Ln(1+stdr_t)$	-2.1 (0.58)	-2.1 (0.58)	-2.5 (0.48)
Negative Price Shock $_{t-1}$ * $Ln(1+stdr_{t-1})$	-3.6** (0.02)	-3.6** (0.02)	-4.1** (0.012)
Trade	-1.69 (0.39)	-1.86 (0.36)	-0.79 (0.61)
Fixed Exchange Rate		0.90 (0.9)	0.7 (0.30)
Polity2			-0.037 (0.55)

[4] stdr = total reserves/short term debt

Table [A1]

Benchmark Parameters

Parameters for Terms of Trade Shock	Benchmark Value
Size of the Shock, γ_{TOT}	0.61
Output loss due to the TOT shock, γ_1	0.025
Output loss due to the TOT shock, γ_2	0.0036
Coefficient of Risk Aversion, σ	1 [0.5, 2]
Share of Tradable consumption, α	0.5
Probability of TOT Shock, π_{TOT}	0.05
Term Premium, δ	0.03
Potential Output Growth, g	0.05
Risk free Rate of Return, r	0.05
Short Term Debt as a ratio of GDP, λ	0.05
Omega, ϖ_t	0.5
Sigmastar, σ^*	[1.5, 2]
Spread, x_t	0.08